MATHEMATICS

Grade 8

Book 1

CAPS

Learner Book

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Whole numbers

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677	831	912	1 008	1 133	1 249	1 315	1 441	1 562	1 598
743	897	978	1 074	1 199	1 315	1 381	1 507	1 628	1 664
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947	1 101	1 182	1 278	1 403	1 519	1 585	1 711	1 832	1 868

+	3 154	7 235	5 331	9 456	2 572	3 638	4 764	2 885	4 921
228	3 382	7 463	5 559	9 684	2 800	3 866	4 992	3 113	5 149
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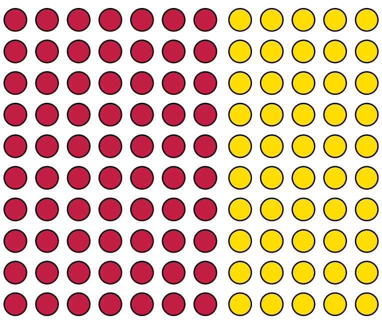
539	3 693	7 774	5 870	9 995	3 111	4 177	5 303	3 424	5 460
677	3 831	7 912	6 008	10 133	3 249	4 315	5 441	3 562	5 598
743	3 897	7 978	6 074	10 199	3 315	4 381	5 507	3 628	5 664
829	3 983	8 064	6 160	10 285	3 401	4 467	5 593	3 714	5 750
947	4 101	8 182	6 278	10 403	3 519	4 585	5 711	3 832	5 868

1 Whole numbers

1.1 Properties of whole numbers

THE COMMUTATIVE PROPERTY OF ADDITION AND MULTIPLICATION



1. Which of the following calculations would you choose to calculate the number of yellow beads in this pattern? Do not do any calculations now, just make a choice.

   (a) 7 + 7 + 7 + 7 + 7

   (b) 10 + 10 + 10 + 10 + 10 + 10 + 10

   (c) 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5

   (d) 5 + 5 + 5 + 5 + 5 + 5 + 5

   (e) 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7

   (f) 10 + 10 + 10 + 10 + 10

   My choice: ....

2. (a) How many red beads are there in the pattern, and how many yellow beads?

   ....

   (b) How many beads are there in the pattern in total? ....

3. (a) Which expression describes what you did to calculate the total number of beads: 70 + 50 or 50 + 70? ....

   (b) Does it make a difference? ....

   (c) Which expression describes what you did to calculate the number of red beads: 7 × 10 or 10 × 7? ....

   (d) Does it make a difference? ....

We say: **addition and multiplication are commutative**. The numbers can be swopped around and their order does not change the answer. This does not work for subtraction and division, however.

4. Calculate each of the following:

5 × 8	10 × 8	12 × 8	8 × 12 ......
6 × 8	3 × 7	6 × 7	7 × 6 ......

THE ASSOCIATIVE PROPERTY OF ADDITION AND MULTIPLICATION

Lebogang and Nathi both have to calculate 25 × 24.

Lebogang calculates 25 × 4 and then multiplies by 6.

Nathi calculates 25 × 6 and then multiplies by 4.

1. Will they get the same answer or not? ......

If three or more numbers have to be multiplied, it does not matter which two of the numbers are multiplied first.

This is called the **associative property of multiplication**. We also say **multiplication is associative.**

2. Do the following calculations. **Do not use a calculator now.**

(a) 4 + 7 + 5 + 6 .... (b) 7 + 6 + 5 + 4 ....

(c) 6 + 5 + 7 + 4 .... (d) 7 + 5 + 4 + 6 ....

3. (a) Is addition associative? ......

(b) Illustrate your answer with an example. ....

4. Find the value of each expression by working in the easiest possible way.

(a) 2 × 17 × 5 .... (b) 4 × 7 × 5 ....

(c) 75 + 37 + 25 .... (d) 60 + 87 + 40 + 13 ....

5. What must you add to each of the following numbers to get 100?

82 44 56 78 24 89 77

....

6. What must you multiply each of these numbers by to get 1 000?

250 125 25 500 200 50

....

7. Calculate each of the following. Note that you can make the work very easy by being smart in deciding how to group the operations.

(a) 82 + 54 + 18 + 46 + 237 (b) 24 + 89 + 44 + 76 + 56 + 11

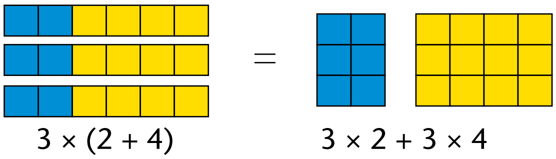
....

(c) 25 × (86 × 4) (d) 32 × 125

....

MORE CONVENTIONS AND THE DISTRIBUTIVE PROPERTY

The distributive property is a useful property because it allows us to do this:



Both answers are 18. Notice that we have to use brackets in the first example to show that the addition operation must be done first. Otherwise, we would have done the multiplication first. For example, the expression $3 \times 2 + 4$ means "multiply 3 by 2; then add 4". It does **not** mean "add 2 and 4; then multiply by 3".

The expression $4 + 3 \times 2$ also means "multiply 3 by 2; then add 4".

> If you wish to specify that addition or subtraction should be **done first**, that part of the expression should be enclosed **in brackets**.

The distributive property can be used to break up a difficult multiplication into smaller parts. For example, it can be used to make it easier to calculate 6×204:

6×204 can be rewritten as $6 \times (200 + 4)$ (Remember the brackets!)

$= 6 \times 200 + 6 \times 4$

$= 1\,200 + 24$

$= 1\,224$

Multiplication can also be distributed over subtraction, for example to calculate 7×96:

$7 \times 96 = 7 \times (100 - 4)$

$= 7 \times 100 - 7 \times 4$

$= 700 - 28$

$= 672$

1. Here are some calculations with answers. Rewrite them with brackets to make all the answers correct.

(a) $8 + 6 \times 5 = 70$ (b) $8 + 6 \times 5 = 38$

....

(c) $5 + 8 \times 6 - 2 = 52$ (d) $5 + 8 \times 6 - 2 = 76$

....

(e) $5 + 8 \times 6 - 2 = 51$ (f) $5 + 8 \times 6 - 2 = 37$

....

2. Calculate the following:

(a) 100 × (10 + 7) (b) 100 × 10 + 100 × 7

....

(c) 100 × (10 – 7) (d) 100 × 10 – 100 × 7

....

3. Complete the table.

×	**8**	**5**	**4**	**9**	**7**	**3**	**6**	**2**	**10**	**11**	**12**
7											
3				27				6			
9											
5											
8											
6											
4					28						
2											
10	80									110	
12											
11											

4. Use the various mathematical conventions for numerical expressions to make these calculations easier. Show all your working.

(a) 18 × 50 (b) 125 × 28 (c) 39 × 220

....

....

....

(d) 443 + 2 100 + 557 (e) 318 + 650 + 322 (f) 522 + 3 003 + 78

....

....

Two more properties of numbers are:

- **The additive property of 0:** when we add zero to any number, the answer is that number.
- **The multiplicative property of 1:** when we multiply any number by 1, the answer is that number.

1.2 Calculations with whole numbers

ESTIMATING, APPROXIMATING AND ROUNDING

1. Try to give answers that you trust to these questions, without doing any calculations with the given numbers.

   (a) Is 8 × 117 more than 2 000 or less than 2 000? .... than 2 000

   ....

   (b) Is 27 × 88 more than 3 000 or less than 3 000? .... than 3 000

   ....

   (c) Is 18 × 117 more than 3 000 or less than 3 000? .... than 3 000

   ....

   (d) Is 47 × 79 more than 3 000 or less than 3 000? .... than 3 000

   ....

What you have done when you tried to give answers to questions 1(a) to (d), is called **estimation**. To estimate is to try to get close to an answer without actually doing the required calculations with the given numbers.

An estimate may also be called an **approximation**.

2. Look at question 1 again.
   (a) The numbers 1 000, 2 000, 3 000, 4 000, 5 000, 6 000, 7 000, 8 000, 9 000 and 10 000 are all multiples of a thousand. In each case, write down the multiple of 1 000 that you think is closest to the answer. Write it on the short dotted line. The numbers you write down are called **estimates**.
   (b) In some cases you may think that you may achieve a **better estimate** by adding 500 to your estimate, or subtracting 500 from it. If so, you may add or subtract 500.
   (c) If you wish, you may write what you believe is an even better estimate by adding or subtracting some hundreds.

3. (a) Use a calculator to find the exact answers for the calculations in question 1, or look up the answers in one of the tables on page 2. Calculate the **error** in your last approximation of each of the answers in question 1.

   The difference between an estimate and the actual answer is called the **error**.

   (b) What was your smallest error? ....

4. Think again about what you did in question 2. In 2(a) you tried to approximate the answers **to the nearest** 1 000. In 2(c) you tried to approximate the answers to the nearest 100. Describe what you tried to achieve in question 2(b).

....

5. Estimate the answers for each of the following products and sums. Try to approximate the answers for the products to the nearest thousand, and for the sums to the nearest hundred. Use the first line in each question to do this.

(a) 84×178 .... (b) $677 + 638$ ....

....

(c) 124×93 .... (d) $885 + 473$ ....

....

(e) 79×84 .... (f) $921 + 367$ ....

....

(g) 56×348 .... (h) $764 + 829$ ....

....

6. Use a calculator to find the exact answers for the calculations in question 5, or look up the answers in the tables on page 2. Calculate the error in each of your approximations. Use the second line in each question to do this.

Calculating with "easy" numbers that are close to given numbers is a good way to obtain approximate answers, for example:

- To approximate $764 + 829$ one may calculate $800 + 800$ to get the approximate answer 1 600, with an error of 7.
- To approximate 84×178 one may calculate 80×200 to get the approximate answer 16 000, with an error of 1 048.

7. Calculate with "easy" numbers close to the given numbers to produce approximate answers for each product below. **Do not use a calculator**. When you have made your approximations, look up the precise answers in the top table on page 2.

(a) 78×46 .... (b) 67×88 ....

....

(c) 34×276 .... (d) 78×178 ....

....

ROUNDING OFF AND COMPENSATING

> The word **compensate** means to do things that will remove damage.

1. (a) Approximate the answer for 386 + 3 435, by rounding both numbers off to the nearest hundred, and adding the rounded numbers.

   ....

   (b) Because you rounded 386 up to 400, you introduced an error of 14 in your approximate answer. What error did you introduce by rounding 3 435 down to 3 400?

   ....

   (c) What was the combined (total) error introduced by rounding both numbers off before calculating?

   ....

   (d) Use your knowledge of the total error to correct your approximate answer, so that you have the correct answer for 386 + 3 435.

   ....

What you have done in question 1 to find the correct answer for 386 + 3 435 is called **rounding off and compensating**. By rounding the numbers off you introduced errors. You then compensated for the errors by making adjustments to your answer.

2. Round off and compensate to calculate each of the following accurately:

(a) 473 + 638	(b) 677 + 921
....	....
....	....

Subtraction can also be done in this way. For example, to calculate R5 362 – R2 687, you may round R2 687 up to R3 000. The calculation can proceed as follows:

- Rounding R2 687 up to R3 000 can be done in two steps: 2 687 + 13 = 2 700, and 2 700 + 300 = 3 000. In total, 313 is added.
- 313 can now be added to 5 362 too: R5 362 + 313 = 5 675.
- Instead of calculating R5 362 – R2 687, which is a bit difficult, you may calculate R5 675 – R3 000. This is easy: R5 675 – R3 000 = R2 675.

This means that R5 362 – R2 687 = R2 675, because
R5 362 – R2 687 = (R5 362 + R313) – (R2 687 + R313).

ADDING NUMBERS IN PARTS WRITTEN IN COLUMNS

Numbers can be added by thinking of their **parts** as we say the numbers.

For example, we say 4 994 as *four thousand nine hundred and ninety-four*.
This can be written in expanded notation as 4 000 + 900 + 90 + 4.

Similarly, we can think of 31 837 as 30 000 + 1 000 + 800 + 30 + 7.

31 837 + 4 994 can be calculated by working with the various kinds of parts separately. To make this easy, the numbers can be written below each other so that the units are below the units, the tens below the tens and so on, as shown on the right.

31 837
4 994

We write only this:	In your mind you can see this:				
31 837	30 000	1 000	800	30	7
4 994		4 000	900	90	4

The numbers in each column can be added to get a new set of numbers.

31 837	30 000	1 000	800	30	7
4 994		4 000	900	90	4
11					11
120				120	
1 700			1 700		
5 000		5 000			
30 000	30 000				
36 831	It is easy to add the new set of numbers to get the answer.				

The work may start with the 10 000s or any other parts. Starting with the units as shown above makes it possible to do more of the work mentally, and write less, as shown below.

31 837 4 994 36 831	To achieve this, only the units digit 1 of the 11 is written in the first step. The 10 of the 11 is remembered and added to the 30 and 90 of the tens column, to get 130.

We say the 10 is **carried** from the units column to the tens column. The same is done when the tens parts are added to get 130: only the digit "3" is written (in the tens column, so it means 30), and the 100 is carried to the next step.

1. Calculate each of the following without using a calculator:

   (a) 4 638 + 2 667 (b) 748 + 7 246

   ....

   ....

   ....

2. Impilo Enterprises plans a new computerised training facility in their existing building. The training manager has to keep the total expenditure budget under R1 million. This is what she has written so far:

Architects and builders	R 102 700
Painting and carpeting	R 42 600
Security doors and blinds	R 52 000
Data projector	R 4 800
25 new secretary chairs	R 50 400
24 desks for work stations	R123 000
1 desk for presenter	R 28 000
25 new computers	R300 000
12 colour laser printers	R 38 980

Work out the total cost of all the items the training manager has budgeted for.

....

....

....

....

....

....

3. Calculate each of the following without using a calculator:

(a) 7 828 + 6 284

....

....

....

....

(b) 7 826 + 888 + 367

....

....

....

....

(c) 657 + 32 890 + 6 542

....

....

....

....

(d) 6 666 + 3 333 + 1

....

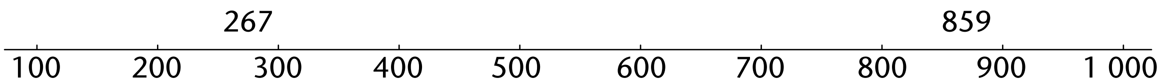
....

....

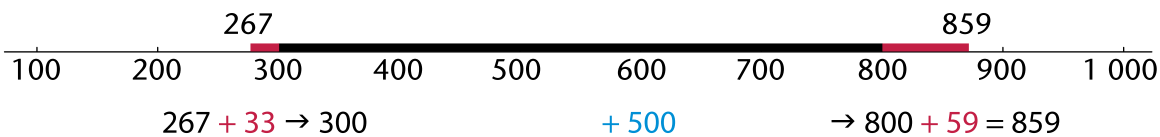
....

METHODS OF SUBTRACTION

There are many ways to find the difference between two numbers. For example, to find the difference between 267 and 859 one may think of the numbers as they may be written on a number line.



We may think of the distance between 267 and 859 as three steps: from 267 to 300, from 300 to 800, and from 800 to 859. How big are each of these three steps?



The above shows that 859 – 267 is 33 + 500 + 59.

1. Calculate 33 + 500 + 59 to find the answer for 859 – 267.

   ....

2. Calculate each of the following. You may think of working out the distance between the two numbers as shown above, or use any other method you prefer.
   Do not use a calculator now.

   (a) 823 – 456 (b) 1 714 – 829

   ....

   ....

   (c) 3 045 – 2 572 (d) 5 131 – 367

   ....

   ....

You can use the tables of sums on page 2 to check your answers for question 2.

Like addition, subtraction can also be done by working with the different parts in which we say numbers. For example, 8 764 – 2 352 can be calculated as follows:

8 thousand – 2 thousand = 6 thousand
7 hundred – 3 hundred = 4 hundred
6 tens – 5 tens = 1 ten
4 units – 2 units = 2 units

So, 8 764 – 2 352 = 6 412

Subtraction by parts is more difficult in some cases, for example 6 213 – 2 758:

6 000 – 2 000 = 4 000. This step is easy, but the following steps cause problems:

200 – 700 = **?**

10 – 50 = **?**

3 – 8 = **?**

Fortunately, the parts and sequence of work may be rearranged to overcome these problems, as shown below:

> One way to overcome these problems is to work with negative numbers:
> 200 – 700 = (–500)
> 10 – 50 = (–40)
> 3 – 8 = (–5)
> 4 000 – 500 → 3 500 – 45 =

instead of		we may do		
3 – 8 =	?	13 – 8 =	....	"borrow" 10 from below
10 – 50 =	?	100 – 50 =	....	"borrow" 100 from below
200 – 700 =	?	1 100 – 700 =	....	"borrow" 1 000 from below
6 000 – 2 000 =	?	5 000 – 2 000 =	....	

This reasoning can also be set out in columns:

instead of				we may do				but write only this			
6 000	200	10	3	5 000	1 100	100	13	6	2	1	3
2 000	700	50	8	2 000	700	50	8	2	7	5	8
				3 000	400	50	5	3	4	5	5

3. (a) Complete the above calculations and find the answer for 6 213 – 2 758.

....

(b) Use the borrowing technique to calculate 823 – 376 and 6 431 – 4 968.

....

....

....

....

....

4. Check your answers in question 3(b) by doing addition.

....

....

....

With some practice, you can learn to subtract using borrowing without writing all the steps. It is convenient to work in columns, as shown on the right for calculating 6 213 – 2 758.

6 213
2 758
5
50
400
3 000
3 455

In fact, by doing more work mentally, you may learn to save more paper by writing even less as shown below.

6 213
2 758
3 455

Do not use a calculator when you do question 5, because the purpose of this work is for you to come to understand methods of subtraction. What you will learn here will later help you to understand **algebra** better.

5. Calculate each of the following:

   (a) 7 342 – 3 877 (b) 8 653 – 1 856 (c) 5 671 – 4 528

You may use a calculator to do questions 6 and 7.

6. Estimate the difference between the two car prices in each case to the nearest R1 000 or closer. Then calculate the difference.

   (a) R102 365 and R98 128 (b) R63 378 and R96 889

7. First estimate the answers to the nearest 100 000 or 10 000 or 1 000. Then calculate.

   (a) 238 769 –141 453 (b) 856 333 – 739 878 (c) 65 244 – 39 427

A METHOD OF MULTIPLICATION

7 × 4 598 can be calculated in parts, as shown here:

$$\begin{aligned} 7 \times 4\,000 &= 28\,000 \\ 7 \times 500 &= 3\,500 \\ 7 \times 90 &= 630 \\ 7 \times 8 &= 56 \end{aligned}$$

$$\begin{array}{rrrrr} & 4 & 5 & 9 & 8 \\ & & & & 7 \\ \hline & & & 5 & 6 \\ & & 6 & 3 & 0 \\ & 3 & 5 & 0 & 0 \\ 2 & 8 & 0 & 0 & 0 \\ \hline 3 & 2 & 1 & 8 & 6 \end{array}$$

The four partial products can now be added to get the answer, which is 32 186. It is convenient to write the work in vertical columns for units, tens, hundreds and so on, as shown on the right.

The answer can be produced with less writing, by "carrying" parts of the partial answers to the next column, when working from right to left in the columns. Only the 6 of the product 7 × 8 is written down instead of 56. The 50 is kept in mind, and added to the 630 obtained when 7 × 90 is calculated in the next step.

$$\begin{array}{rrrrr} & 4 & 5 & 9 & 8 \\ & & & & 7 \\ \hline 3 & 2 & 1 & 8 & 6 \end{array}$$

1. Calculate each of the following. Do not use a calculator now.

   (a) 27 × 649 (b) 75 × 1 756 (c) 348 × 93

2. Use your calculator to check your answers for question 1. Redo the questions for which you had the wrong answers.

3. Calculate each of the following. Do not use a calculator now.

   (a) 67 × 276 (b) 84 × 178

4. Use the product table on page 2 or a calculator to check your answers for question 3. Redo the questions for which you had the wrong answers.

LONG DIVISION

1. The municipal head gardener wants to buy young trees to plant along the main street of the town. The young trees cost R27 each, and an amount of R9 400 has been budgeted for trees. He needs 324 trees. Do you think he has enough money?

   ....

2. (a) How much will 300 trees cost? ....
   (b) How much money will be left if 300 trees are bought? ....
   (c) How much money will be left if 20 more trees are bought? ....

The municipal gardener wants to work out exactly how many trees, at R27 each, he can buy with the budgeted amount of R9 400. His thinking and writing are described below.

Step 1

What he writes:	What he thinks:
27	9 400	*I want to find out how many chunks of 27 there are in 9 400.*

Step 2

What he writes:	What he thinks:
300	*I think there are at least 300 chunks of 27 in 9 400.*
27	9 400	
$\underline{8\ 100}$	*300 × 27 = 8 100. I need to know how much is left over.*
1 300	*I want to find out how many chunks of 27 there are in 1 300.*

Step 3 (He has to rub out the one "0" of the 300 on top, to make space.)

What he writes:	What he thinks:
340	*I think there are at least 40 chunks of 27 in 1 300.*
27	9 400	
$\underline{8\ 100}$	
1 300	
$\underline{1\ 080}$	*40 × 27 = 1 080. I need to know how much is left over.*
220	*I want to find out how many chunks of 27 there are in 220.*
	Perhaps I can buy some extra trees.

Step 4 (He rubs out another "0".)

What he writes:	What he thinks:
348	*I think there are at least 8 chunks of 27 in 220.*
27	9 400	
$\underline{8\ 100}$	
1 300	
$\underline{1\ 080}$	
220	
$\underline{216}$	*8 × 27 = 216*
4	*So, I can buy 348 young trees and will have R4 left.*

Do not use a calculator to do questions 3 and 4. The purpose of this work is for you to develop a good understanding of how division can be done. Check all your answers by doing multiplication.

3. (a) Graham bought 64 goats, all at the same price. He paid R5 440 in total. What was the price for each goat? Your first step can be to work out how much he would have paid if he paid R10 per goat, but you can start with a bigger step if you wish.

   ....
   ....
   ....
   ....
   ....

   (b) Mary has R2 850 and she wants to buy candles for her sister's wedding reception. The candles cost R48 each. How many candles can she buy?

   ....
   ....
   ....
   ....
   ....
   ....
   ....

4. Calculate each of the following, without using a calculator:

   (a) 7 234 ÷ 48

   ....
   ....
   ....
   ....
   ....
   ....

   (b) 3 267 ÷ 24

   ....
   ....
   ....
   ....
   ....
   ....

   (c) 9 500 ÷ 364

   ....
   ....
   ....
   ....
   ....
   ....
   ....

   (d) 8 347 ÷ 24

   ....
   ....
   ....
   ....
   ....
   ....
   ....

1.3 Multiples, factors and prime factors

MULTIPLES AND FACTORS

1. The numbers 6; 12; 18; 24; ... are **multiples** of 6.
   The numbers 7; 14; 21; 28; ... are **multiples** of 7.

> If n is a natural number, $6n$ represents the multiples of 6.

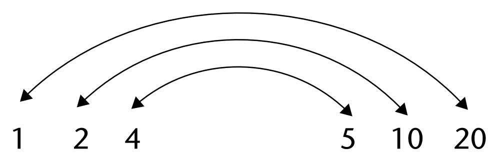
   (a) What is the 100th number in each sequence above?
   (b) Is 198 a number in the first sequence?
   (c) Is 175 a number in the second sequence?

Of which numbers is 20 a multiple?

$20 = 1 \times 20 = 2 \times 10 = 4 \times 5 = 5 \times 4 = 10 \times 2 = 20 \times 1$

> 20 is a multiple of 1; 2; 4; 5; 10 and 20 and all of these numbers are factors of 20.

Factors come in pairs. The following pairs are factors of 20:



2. A rectangle has an area of 30 cm. What are the possible lengths of the sides of the rectangle in centimetres if the lengths of the sides are natural numbers?

3. Are 4; 8; 12 and 16 factors of 48? Simon says that all multiples of 4 smaller than 48 are factors of 48. Is he right?

4. We have defined factors in terms of the product of *two* numbers. What happens if we have a product of *three* or more numbers, for example $210 = 2 \times 3 \times 5 \times 7$?
   (a) Explain why 2; 3; 5 and 7 are factors of 210.
   (b) Are 2×3; 3×5; 5×7; 2×5 and 2×7 factors of 210?
   (c) Are $2 \times 3 \times 5$; $3 \times 5 \times 7$ and $2 \times 5 \times 7$ factors of 210?

5. Is 20 a factor of 60? What factors of 20 are also factors of 60?

PRIME NUMBERS AND COMPOSITE NUMBERS

The number 36 can be formed as $2 \times 2 \times 3 \times 3$. Because 2 and 3 are used twice, they are called **repeated factors** of 36.

1. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.

   (a) 66 .... (b) 67 ....

   (c) 68 .... (d) 69 ....

   (e) 70 .... (f) 71 ....

   (g) 72 ....
   .... (h) 73 ....
   ....

2. Which of the numbers in question 1 cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*? ....

A number that cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*, is called a **prime number**.

3. Which of the numbers in question 1 are prime? ....

Composite numbers are natural numbers with more than two different factors. The sequence of composite numbers is 4; 6; 8; 9; 10; 12; ...

4. Are the statements below true or false? If you answer "false", explain why.

   (a) All prime numbers are odd numbers.

   ....

   (b) All composite numbers are even numbers.

   ....

   (c) 1 is a prime number.

   ....

   (d) If a natural number is not prime, then it is composite.

   ....

(e) 2 is a composite number.

....

(f) 785 is a prime number.

....

(g) A prime number can only end in 1; 3; 7 or 9.

....

(h) Every composite number is divisible by at least one prime number.

....

5. We can find out whether a given number is prime by systematically checking whether the primes 2; 3; 5; 7; 11; 13; ... are factors of the given number or not.
   To find possible factors of 131, we need to consider only the primes 2; 3; 5; 7 and 11. Why not 13; 17; 19; ...?

   ....

6. Determine whether the following numbers are prime or composite. If the number is composite, write down at least two factors of the number (besides 1 and the number itself).

(a) 221	(b) 713
....	....
....	....

PRIME FACTORISATION

Every composite number can be expressed as the product of prime factors and this can happen in only one way.

To find all the factors of a number you can write the number as the product of prime factors, first by writing it as the product of two convenient (composite) factors and then by splitting these factors into smaller factors until all factors are prime. Then you take all the possible combinations of the products of the prime factors.

Example: Find the factors of 84.

Write 84 as the product of prime factors by starting with different known factors:

$84 = 4 \times 21$	or	$84 = 7 \times 12$	or	$84 = 2 \times 42$
$= 2 \times 2 \times 3 \times 7$		$= 7 \times 3 \times 4$		$= 2 \times 6 \times 7$
		$= 7 \times 3 \times 2 \times 2$		$= 2 \times 2 \times 3 \times 7$

A **more systematic way** of finding the prime factors of a number would be to start with the prime numbers and try the consecutive prime numbers 2; 3; 5; 7; … as possible factors. The work may be set out as shown below.

$$\begin{array}{r|r} 2 & 1\,430 \\ \hline 5 & 715 \\ \hline 11 & 143 \\ \hline 13 & 13 \\ \hline & 1 \end{array} \qquad\qquad \begin{array}{r|r} 3 & 2\,457 \\ \hline 3 & 819 \\ \hline 3 & 273 \\ \hline 7 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$1\,430 = 2 \times 5 \times 11 \times 13$ $\qquad$ $2\,457 = 3 \times 3 \times 3 \times 7 \times 13$

We can use exponents to write the products of prime factors more compactly as products of powers of prime factors.

$$\begin{aligned} 2\,457 &= 3 \times 3 \times 3 \times 7 \times 13 &&= 3^3 \times 7 \times 13 \\ 72 &= 2 \times 2 \times 2 \times 3 \times 3 &&= 2^3 \times 3^2 \\ 1\,500 &= 2 \times 2 \times 3 \times 5 \times 5 \times 5 &&= 2^2 \times 3 \times 5^3 \end{aligned}$$

1. Express the following numbers as the product of powers of primes:

   (a) 792 = ... $\qquad$ (b) 444 = ...

2. Find the prime factors of the numbers below.

$$\begin{array}{r|l} 2 & 28 \\ \hline & 14 \\ \hline & \\ \hline & \end{array} \qquad 32 \qquad 124 \qquad 36 \qquad 42 \qquad 345 \qquad 182$$

....

....

COMMON MULTIPLES AND FACTORS

1. Is 4×5 a multiple of 4? .... Is 4×5 a multiple of 5? ....
2. Comment on the following statement:
   The product of numbers is a multiple of each of the numbers in the product.

   ....

We use **common multiples** when fractions with different denominators are added. To add $\frac{2}{3} + \frac{3}{4}$, the common denominator is 3×4, so the sum becomes $\frac{8}{12} + \frac{9}{12}$.

In the same way, we could use $6 \times 8 = 48$ as a common denominator to add $\frac{1}{6} + \frac{3}{8}$, but 24 is the **lowest common multiple** (**LCM**) of 6 and 8.

Prime factorisation makes it easy to find the lowest common multiple or highest common factor.

When we simplify a fraction, we divide the same number into the numerator and the denominator. For the simplest fraction, use the **highest common factor** (**HCF**) to divide into both numerator and denominator.

The HCF is divided into the numerator and the denominator to write the fraction in its **simplest form**.

$$\text{So } \frac{36}{144} = \frac{2 \times 2 \times 3 \times 3}{2 \times 2 \times 2 \times 2 \times 3 \times 3} = \frac{1}{4}$$

Use prime factorisation to determine the LCM and HCF of 32, 48 and 84 in a systematic way:

$$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$$
$$84 = 2 \times 2 \times 3 \times 7 \quad = 2^2 \times 3 \times 7$$

The LCM is a **multiple**, so all of the factors of all the numbers must divide into it.

All of the factors that are present in the three numbers must also be factors of the LCM, even if it is a factor of only one of the numbers. But because it has to be the lowest common multiple, no unnecessary factors are in the LCM.

The highest power of each factor is in the LCM, because then all of the other factors can divide into it. In 32, 48 and 84, the highest power of 2 is 2^5, the highest power of 3 is 3 and the highest power of 7 is 7.

LCM = $2^5 \times 3 \times 7 = 672$

The HCF is a common factor. Therefore, for a factor to be in the HCF, it must be a factor of *all* of the numbers. 2 is the only number that appears as a factor of all three numbers. The lowest power of 2 is 2^2, so the HCF is 2^2.

3. Determine the LCM and the HCF of the numbers in each case.

(a) 24; 28; 42

....
....
....
....
....

(b) 17; 21; 35

....
....
....
....
....

(c) 75; 120; 200

....
....
....
....
....

(d) 18; 30; 45

....
....
....
....
....

INVESTIGATE PRIME NUMBERS

You may use a calculator for this investigation.

1. Find all the prime numbers between 110 and 130.

2. Find all the prime numbers between 210 and 230.

3. Find the biggest prime number smaller than 1 000.

1.4 Solving problems

RATE AND RATIO

You may use a calculator for the work in this section.

1. Tree plantations in the Western Cape are to be cut down in favour of natural vegetation. There are roughly 3 000 000 trees on plantations in the area and it is possible to cut them down at a **rate** of 15 000 trees per day with the labour available. How many working days will it take before all the trees will be cut down?

....

....

Instead of saying "... per day", people often say "at a **rate** of ... per day". Speed is a way to describe the rate of movement.

The word **per** is often used to describe a rate and can mean *for every*, *for*, *in each*, *in*, *out of*, or *every*.

2. A car travels a distance of 180 km in 2 hours on a straight road. How many kilometres can it travel in 3 hours at the same speed?

....

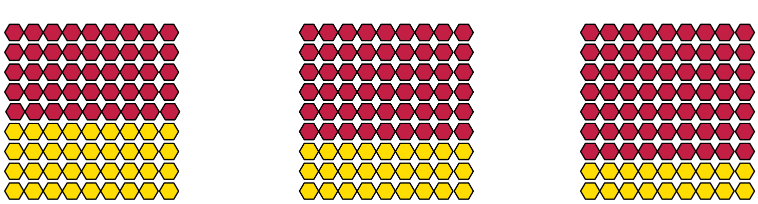
3. Thobeka wants to order a book that costs \$56,67. The rand-dollar exchange rate is R7,90 to a dollar. What is the price of the book in rands?

....

....

....

4. In pattern A below, there are 5 red beads for every 4 yellow beads.



Pattern A *Pattern B* *Pattern C*

Describe patterns B and C in the same way.

....

....

5. Complete the table to show how many screws are produced by two machines in different periods of time.

Number of hours	1	2	3	5	8
Number of screws at machine A	1 800				
Number of screws at machine B	2 700				

(a) How much faster is machine B than machine A? ....

(b) How many screws will machine B produce in the same time that it takes machine A to make 100 screws?

....

The patterns in question 4 can be described like this: In pattern A, the **ratio** of yellow beads to red beads is 4 to 5. This is written as 4 : 5. In pattern B, the ratio between yellow beads and red beads is 3 : 6. In pattern C the ratio is 2 : 7.

In question 5, machine A produces 2 screws for every 3 screws that machine B produces. This can be described by saying that the ratio between the production speeds of machines A and B is 2 : 3.

6. Nathi, Paul and Tim worked in Mr Setati's garden. Nathi worked for 5 hours, Paul for 4 hours and Tim for 3 hours. Mr Setati gave the boys R600 for their work. How should they divide the R600 among the three of them?

....

....

We use **ratios** to show how many times more, or less, one quantity is than another.

A **ratio** is a comparison of two (or more) quantities. The number of hours that Nathi, Paul and Tim worked are in the ratio 5 : 4 : 3. To be fair, the money should also be shared in that ratio. That means that Nathi should receive 5 parts, Paul 4 parts and Tim 3 parts of the money. There were 12 parts, which means Nathi should receive $\frac{5}{12}$ of the total amount, Paul should get $\frac{4}{12}$ and Tim should get $\frac{3}{12}$.

7. Ntabi uses 3 packets of jelly to make a pudding for 8 people. How many packets of jelly does she need to make a pudding for 16 people? And for 12 people?

....

8. Which rectangle is more like a square: a 3 × 5 rectangle or a 6 × 8 rectangle? Explain.

....

To increase 40 in the ratio 2 : 3 means that the 40 represents two parts and must be increased so that the new number represents 3 parts. If 40 represents two parts, 20 represents 1 part. The increased number will therefore be 20 × 3 = 60.

Remember that if you multiply by 1, the number does not change. If you multiply by a number greater than 1, the number increases. If you multiply by a number smaller than 1, the number decreases.

9. (a) Increase 56 in the ratio 2 : 3.

....

(b) Decrease 72 in the ratio 4 : 3.

....

10. (a) Divide 840 in the ratio 3 : 4.

....

(b) Divide 360 in the ratio 1 : 2 : 3.

....

11. Some data about the performance of different athletes during a walking event is given below. Investigate the data to find out who walks fastest and who walks slowest. Arrange the athletes from the fastest walker to the slowest walker.

(a) First make estimates to do the investigation.

(b) Then use your calculator to do the investigation.

Athlete	A	B	C	D	E	F
Distance walked in m	2 480	4 283	3 729	6 209	3 112	5 638
Time taken in minutes	17	43	28	53	24	45

....

....

....

....

....

....

....

PROFIT, LOSS, DISCOUNT AND INTEREST

1. (a) How much is 1 eighth of R800? ....

   (b) How much is 1 hundredth of R800? ....

   (b) How much is 7 hundredths of R800? ....

Rashid is a furniture dealer. He buys a couch for R2 420. He displays the couch in his showroom with the price marked as R3 200. Rashid offers a discount of R320 to customers who pay cash.

> The amount for which a dealer buys an article from a producer or manufacturer is called the **cost price**. The price marked on the article is called the **marked price** and the price of the article after discount is the **selling price**.

2. (a) What is the cost price of the couch in Rashid's furniture shop? ....

   (b) What is the marked price? ....

   (c) What is the selling price for a customer who pays cash? ....

   (d) How much is 10 hundredths of R3 200? ....

The **discount** on an article is always less than the marked price of the article. In fact, it is only a fraction of the marked price. The discount of R320 that Rashid offers on the couch is 10 hundredths of the marked price.

Another word for hundredths is **percentage**, and the symbol for percentage is %. So we can say that Rashid offers a discount of 10%.

A percentage is a number of hundredths.

18% is 18 hundredths, and 25% is 25 hundredths.

> % is a symbol for hundredths. 8% means 8 hundredths and 15% means 15 hundredths.
> The symbol % is just a variation of the $\frac{\quad}{100}$ that is used in the common fraction notation for hundredths.
> 8% is $\frac{8}{100}$.

A discount of 6% on an article can be calculated in two steps:

Step 1: Calculate 1 hundredth of the marked price (divide by 100).

Step 2: Calculate 6 hundredths of the marked price (multiply by 6).

3. Calculate a discount of 6% on each of the following marked prices of articles:

   (a) R3 600 (b) R9 360

   ....

   ....

4. (a) How much is 1 hundredth of R700? ....

   (b) A customer pays cash for a coat marked at R700.
   He is given R63 discount. How many hundredths of R700 is this? ....

   (c) What is the percentage discount? ....

5. A client buys a blouse marked at R300 and she is given R36 discount for paying cash. Work as in question 4 to determine what percentage discount she was given.

   ....

   ....

You may use a calculator to do questions 6, 7 and 8.

6. A dealer buys an article for R7 500 and makes the price 30% higher. The article is sold at a 20% discount.

   (a) What is the selling price of the article?

   ....

   ....

   (b) What is the dealer's percentage profit?

   ....

When a person borrows money from a bank or some other institution, he or she normally has to pay for the use of the money. This is called **interest**.

7. Sam borrows R7 000 from a bank at 14% interest for one year. How much does he have to pay back to the bank at the end of the period?

   ....

8. Jabu invests R5 600 for one year at 8% interest.

   (a) What will the value of his investment be at the end of that year?

   ....

   (b) At the end of the year Jabu does not withdraw the investment or the interest earned, but reinvests it for another year. How much will it be worth at the end of the second year?

   ....

   ....

   (c) What will the value of Jabu's investment be after five years?

   ....

CHAPTER 2
Integers

In this chapter you will work with whole numbers smaller than 0. These numbers are called negative numbers. The whole numbers larger than 0, 0 itself and the negative whole numbers together are called the **integers**. Mathematicians have agreed that negative numbers should have certain properties that would make them useful for various purposes. You will learn about these properties and how they make it possible to do calculations with negative numbers.

Do what you can.

$5 - 0 = ?$	$5 - 7 = ?$	$5 + 5 = ?$
$5 - 1 = ?$	$5 - 6 = ?$	$5 + 4 = ?$
$5 - 2 = ?$	$5 - 5 = ?$	$5 + 3 = ?$
$5 - 3 = ?$	$5 - 4 = ?$	$5 + 2 = ?$
$5 - 4 = ?$	$5 - 3 = ?$	$5 + 1 = ?$
$5 - 5 = ?$	$5 - 2 = ?$	$5 + 0 = ?$
$5 - 6 = ?$	$5 - 1 = ?$	$5 + ? = ?$
$5 - 7 = ?$	$5 - 0 = ?$	$5 + ? = ?$
$5 - 8 = ?$	$5 - ? = ?$	$5 + ? = ?$
$5 - 9 = ?$	$5 - ? = ?$	$5 + ? = ?$
$5 - 10 = ?$	$5 - ? = ?$	$5 + ? = ?$

Choose a good plan to complete this table.

Plan A: Look at where the number 4 appears in the table. All the 4s lie on a diagonal, going down from left to right. Complete the other diagonals in the same way.

Plan B: Look at the number 13 in the table. It is on the right, in the fourth row from the top. It can be obtained by adding the two numbers indicated by arrows. Complete all the cells by adding numbers from the yellow column and blue row in this way.

Plan C: In each row, add 1 to go right and subtract 1 to go left.

Plan D: In each column, add 1 to go up and subtract 1 to go down.

0	1	2	3	4	5	6	7	**8**	9	10	11	12	13	14	15	16
					4			**7**								
						4		**6**								
–3	–2	–1	0				4	**5**								13
								4								
–5	–4							**3**	4							
								2		4						
								1			4					
–8	**–7**	**–6**	**–5**	**–4**	**–3**	**–2**	**–1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
								–1					4			
								–2						4		
								–3							4	
								–4								4
								–5								
								–6								
								–7							0	
								–8								0

2 Integers

2.1 What is beyond 0?

WHY PEOPLE DECIDED TO HAVE NEGATIVE NUMBERS

On the right, you can see how Jimmy prefers to work when doing calculations such as 542 + 253.

500 + 200 = 700
40 + 50 = 90
2 + 3 = 5
700 + 90 + 5 = 795

He tries to calculate 542 – 253 in a similar way:

500 – 200 = 300
40 – 50 = ?

Jimmy clearly has a problem. He reasons as follows:

I can subtract 40 from 40; that gives 0. But then there is still 10 that I have to subtract.

He decides to deal with the 10 that he still has to subtract later, and continues:

500 – 200 = 300
40 – 50 = 0, *but there is still 10 that I have to subtract.*
2 – 3 = 0, *but there is still 1 that I have to subtract.*

1. (a) What must Jimmy still subtract, and what will his final answer be?

....

....

(b) When Jimmy did another subtraction problem, he ended up with this writing at one stage:

600 and (–)50 and (–)7

What do you think is Jimmy's final answer for this subtraction problem?

....

About 500 years ago, some mathematicians proposed that a "negative number" may be used to describe the result in a situation such as in Jimmy's subtraction problem above, where a number is subtracted from a number smaller than itself.

For example, we may say 10 – 20 = (–10)

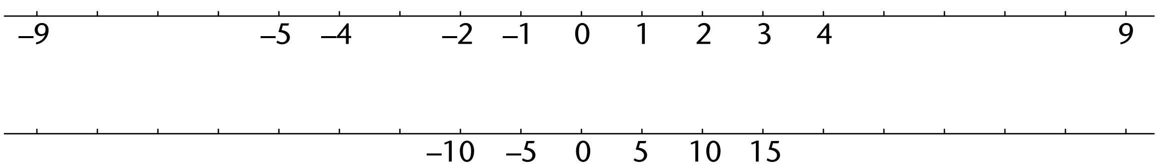
This proposal was soon accepted by other mathematicians, and it is now used all over the world.

Mathematicians are people who do mathematics for a living. Mathematics is their profession, like health care is the profession of nurses and medical doctors.

2. Calculate each of the following:

(a) 16 – 20 ....
(b) 16 – 30 ....
(c) 16 – 40 ....
(d) 16 – 60 ....
(e) 16 – 200 ....
(f) 5 – 1 000 ....

3. Some numbers are shown on the lines below. Fill in the missing numbers.



The following statement is true if the number is 5:

15 – (a certain number) = 10

A few centuries ago, some mathematicians decided they wanted to have numbers that will also make sentences like the following true:

15 + (a certain number) = 10

But to go from 15 to 10 you have to subtract 5.

The numbers 1; 2; 3; 4 etc. are called the **natural numbers**. The natural numbers, 0 and the negative whole numbers together are called the **integers**.

The number we need to make the sentence *15 + (a certain number) = 10* true must have the following strange property:

If you **add** this number, it should have the **same effect** as to **subtract 5**.

Now the mathematicians of a few centuries ago really wanted to have numbers for which such strange sentences would be true. So they thought:

Let us decide, and agree amongst ourselves, that the number we call negative 5 *will have the property that if you add it to another number, the effect will be the same as when you subtract the natural number 5.*

This means that the mathematicians agreed that 15 + (–5) is equal to 15 – 5.

Stated differently, instead of adding *negative* 5 to a number, you may subtract 5.

Adding a negative number has the same effect as subtracting a natural number.
For example: 20 + (–15) = 20 – 15 = 5

4. Calculate each of the following:

(a) 500 + (–300) ....
(b) 100 + (–20) + (–40) ....
(c) 500 + (–200) + (–100) ....
(d) 100 + (–60) ....

5. Make a suggestion of what the answer for (–20) + (–40) should be. Give reasons for your suggestion.

....

6. Continue the lists of numbers below to complete the table.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
10	100	3	–3	–20	150	0
9	90	6	–6	–18	125	–5
8	80	9	–9	–16	100	–10
7	70	12	–12	–14	75	–15
6	60	15	–15		50	–20
5	50					–25
4	40					
3	30					
2	20					
1	10					
–1	–10					

The following statement is true if the number is 5:

15 + (a certain number) = 20

What properties should a number have so that it makes the following statement true?

15 – (a certain number) = 20

To go from 15 to 20 you have to add 5. The number we need to make the sentence *15 – (a certain number)* = 20 true must have the following property:

If you **subtract** this number, it should have the **same effect** as to **add 5**.

Let us agree that 15 – (–5) is equal to 15 + 5.

Stated differently, instead of subtracting *negative 5* from a number, you may add 5.

Subtracting a negative number has the same effect as adding a natural number.
For example: 20 – (–15) = 20 + 15 = 35

7. Calculate.

(a) 30 – (–10)	....	(b) 30 + 10	....
(c) 30 + (–10)	....	(d) 30 – 10	....
(e) 30 – (–30)	....	(f) 30 + 30	....
(g) 30 + (–30)	....	(h) 30 – 30	....

You probably agree that

5 + (–5) = 0 10 + (–10) = 0 and 20 + (–20) = 0

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (–3), are called **additive inverses**. They wipe each other out when you add them.

When you add any number to its additive inverse, the answer is 0 (the additive property of 0). For example, 120 + (–120) = 0.

What may each of the following be equal to?
(–8) + 5
(–5) + (–8)

8. Write the additive inverse of each of the following numbers:

(a) 24	....	(b) –24	....
(c) –103	....	(d) 2 348	....

The idea of additive inverses may be used to explain why 8 + (–5) is equal to 3:

$$8 + (-5) = 3 + \boxed{5 + (-5)} = 3 + 0 = 3$$

9. Use the idea of additive inverses to explain why each of these statements is true:
(a) 43 + (–30) = 13 (b) 150 + (–80) = 70

....

STATEMENTS THAT ARE TRUE FOR MANY DIFFERENT NUMBERS

For how many different pairs of numbers can the following statement be true, if only natural (positive) numbers are allowed?

a number + another number = 10

For how many different pairs of numbers can the statement be true if negative numbers are also allowed?

2.2 Adding and subtracting with integers

ADDING CAN MAKE LESS AND SUBTRACTION CAN MAKE MORE

1. Calculate each of the following:

(a) $10 + 4 + (-4)$ ....
(b) $10 + (-4) + 4$ ....
(c) $3 + 8 + (-8)$ ....
(d) $3 + (-8) + 8$ ....

Natural numbers can be arranged in any order to add and subtract them. This is also the case for integers.

The numbers 1; 2; 3; 4; etc. that we use to count, are called **natural numbers**.

2. Calculate each of the following:

(a) $18 + 12$ ....
(b) $12 + 18$ ....
(c) $2 + 4 + 6$ ....
(d) $6 + 4 + 2$ ....
(e) $2 + 6 + 4$ ....
(f) $4 + 2 + 6$ ....
(g) $4 + 6 + 2$ ....
(h) $6 + 2 + 4$ ....
(i) $6 + (-2) + 4$ ....
(j) $4 + 6 + (-2)$ ....
(k) $4 + (-2) + 6$ ....
(l) $(-2) + 4 + 6$ ....
(m) $6 + 4 + (-2)$ ....
(n) $(-2) + 6 + 4$ ....
(o) $(-6) + 4 + 2$ ....

3. Calculate each of the following:

(a) $(-5) + 10$ ....
(b) $10 + (-5)$ ....
(c) $(-8) + 20$ ....
(d) $20 - 8$ ....
(e) $30 + (-10)$ ....
(f) $30 + (-20)$ ....
(g) $30 + (-30)$ ....
(h) $10 + (-5) + (-3)$ ....
(i) $(-5) + 7 + (-3) + 5$ ....
(j) $(-5) + 2 + (-7) + 4$ ....

4. In each case, find the number that makes the statement true. Give your answer by writing a closed number sentence.

(a) 20 + (an unknown number) = 50

....

(b) 50 + (an unknown number) = 20

....

(c) 20 + (an unknown number) = 10

....

Statements like these are also called number sentences.
An incomplete number sentence, where some numbers are not known at first, is sometimes called an **open number sentence**:

8 – (a number) = 10

A **closed number sentence** is where all the numbers are known:

8 + 2 = 10

(d) (an unknown number) + (–25) = 50

....

(e) (an unknown number) + (–25) = –50

....

5. Use the idea of additive inverses to explain why each of the following statements is true:

(a) 43 + (–50) = –7 ....

(b) 60 + (–85) = –25 ....

6. Complete the table as far as you can.

(a)	(b)	(c)
5 – 8 =	5 + 8 =	8 – 3 =
5 – 7 =	5 + 7 =	7 – 3 =
5 – 6 =	5 + 6 =	6 – 3 =
5 – 5 =	5 + 5 =	5 – 3 =
5 – 4 =	5 + 4 =	4 – 3 =
5 – 3 =	5 + 3 =	3 – 3 =
5 – 2 =	5 + 2 =	2 – 3 =
5 – 1 =	5 + 1 =	1 – 3 =
5 – 0 =	5 + 0 =	0 – 3 =
5 – (–1) =	5 + (–1) =	(–1) – 3 =
5 – (–2) =	5 + (–2) =	(–2) – 3 =
5 – (–3) =	5 + (–3) =	(–3) – 3 =
5 – (–4) =	5 + (–4) =	(–4) – 3 =
5 – (–5) =	5 + (–5) =	(–5) – 3 =
5 – (–6) =	5 + (–6) =	(–6) – 3 =

7. Calculate.

(a) $80 + (-60)$ (b) $500 + (-200) + (-200)$

....

....

8. (a) Is $100 + (-20) + (-20) = 60$, or does it equal something else? ....

(b) What do you think $(-20) + (-20)$ is equal to? ....

9. Calculate.

(a) $20 - 20$ .... (b) $50 - 20$ ....

(c) $(-20) - (-20)$ .... (d) $(-50) - (-20)$ ....

10. Calculate.

(a) $20 - (-10)$ .... (b) $100 - (-100)$ ....

(c) $20 + (-10)$ .... (d) $100 + (-100)$ ....

(e) $(-20) - (-10)$ .... (f) $(-100) - (-100)$ ....

(g) $(-20) + (-10)$ .... (h) $(-100) + (-100)$ ....

11. Complete the table as far as you can.

(a)	(b)	(c)
$5 - (-8) =$	$(-5) + 8 =$	$8 - (-3) =$
$5 - (-7) =$	$(-5) + 7 =$	$7 - (-3) =$
$5 - (-6) =$	$(-5) + 6 =$	$6 - (-3) =$
$5 - (-5) =$	$(-5) + 5 =$	$5 - (-3) =$
$5 - (-4) =$	$(-5) + 4 =$	$4 - (-3) =$
$5 - (-3) =$	$(-5) + 3 =$	$3 - (-3) =$
$5 - (-2) =$	$(-5) + 2 =$	$2 - (-3) =$
$5 - (-1) =$	$(-5) + 1 =$	$1 - (-3) =$
$5 - 0 =$	$(-5) + 0 =$	$0 - (-3) =$
$5 - 1 =$	$(-5) + (-1) =$	$(-1) - (-3) =$
$5 - 2 =$	$(-5) + (-2) =$	$(-2) - (-3) =$
$5 - 3 =$	$(-5) + (-3) =$	$(-3) - (-3) =$
$5 - 4 =$	$(-5) + (-4) =$	$(-4) - (-3) =$
$5 - 5 =$	$(-5) + (-5) = -$	$(-5) - (-3) =$

12. In each case, state whether the statement is true or false and give a numerical example to demonstrate your answer.

(a) Subtracting a positive number from a negative number has the same effect as adding the additive inverse of the positive number.

....

(b) Adding a negative number to a positive number has the same effect as adding the additive inverse of the negative number.

....

(c) Subtracting a negative number from a positive number has the same effect as subtracting the additive inverse of the negative number.

....

(d) Adding a negative number to a positive number has the same effect as subtracting the additive inverse of the negative number.

....

(e) Adding a positive number to a negative number has the same effect as adding the additive inverse of the positive number.

....

(f) Adding a positive number to a negative number has the same effect as subtracting the additive inverse of the positive number.

....

(g) Subtracting a positive number from a negative number has the same effect as subtracting the additive inverse of the positive number.

....

(h) Subtracting a negative number from a positive number has the same effect as adding the additive inverse of the negative number.

....

COMPARING INTEGERS AND SOLVING PROBLEMS

1. Fill <, > or = into the block to make the relationship between the numbers true:

   (a) –103 ☐ –99

   (b) –699 ☐ –701

   (c) 30 ☐ –30

   (d) 10 – 7 ☐ –(10 – 7)

   (e) –121 ☐ –200

   (f) 12 – 5 ☐ –(12 + 5)

   (g) –199 ☐ –110

2. At 5 a.m. in Bloemfontein the temperature was –5 °C. At 1 p.m., it was 19 °C. By how many degrees did the temperature rise?

   ....

3. A diver swims 150 m below the surface of the sea. She moves 75 m towards the surface. How far below the surface is she now?

   ....

   ....

4. One trench in the ocean is 800 m deep and another is 2 200 m deep. What is the difference in their depths?

   ....

5. An island has a mountain which is 1 200 m high. The surrounding ocean has a depth of 860 m. What is the difference in height?

   ....

6. On a winter's day in Upington the temperature rose by 19 °C. If the minimum temperature was –4 °C, what was the maximum temperature?

   ....

2.3 Multiplying and dividing with integers

MULTIPLICATION WITH INTEGERS

1. Calculate.
   (a) $-5 + -5 + -5 + -5 + -5 + -5 + -5 + -5 + -5 + -5$ ....
   (b) $-10 + -10 + -10 + -10 + -10$ ....
   (c) $-6 + -6 + -6 + -6 + -6 + -6 + -6 + -6$ ....
   (d) $-8 + -8 + -8 + -8 + -8 + -8$ ....
   (e) $-20 + -20 + -20 + -20 + -20 + -20 + -20$ ....

2. In each case, show whether you agree (✓) or disagree (✗) with the given statement.
   (a) $10 \times (-5) = 50$ .... (b) $8 \times (-6) = (-8) \times 6$ ....
   (c) $(-5) \times 10 = 5 \times (-10)$ .... (d) $6 \times (-8) = -48$ ....
   (e) $(-5) \times 10 = 10 \times (-5)$ .... (f) $8 \times (-6) = 48$ ....
   (g) $4 \times 12 = -48$ .... (h) $(-4) \times 12 = -48$ ....

Multiplication of integers is commutative:
$(-20) \times 5 = 5 \times (-20)$

3. Is addition of integers commutative?
   Demonstrate your answer with three different examples.

   ....

4. Calculate.
   (a) $20 \times (-10)$ .... (b) $(-5) \times 4$ ....
   (c) $(-20) \times 10$ .... (d) $4 \times (-25)$ ....
   (e) $29 \times (-20)$ .... (f) $(-29) \times (-2)$ ....

5. Calculate.
   (a) $10 \times 50 + 10 \times (-30)$ (b) $50 + (-30)$

   ....

   (c) $10 \times \mathbf{(}50 + (-30)\mathbf{)}$ (d) $(-50) + (-30)$

   ....

   (e) $10 \times (-50) + 10 \times (-30)$ (f) $10 \times \mathbf{(}(-50) + (-30)\mathbf{)}$

   ....

The product of two positive numbers is a positive number, for example $5 \times 6 = 30$.

The product of a positive number and a negative number is a negative number, for example $5 \times (-6) = -30$.

The product of a negative number and a positive number is a negative number, for example $(-5) \times 6 = -30$.

6. (a) Four numerical expressions are given below. Underline the expressions that you would expect to have the same answers. Do not do the calculations.

   $14 \times (23 + 58)$ $\quad$ $23 \times (14 + 58)$ $\quad$ $14 \times 23 + 14 \times 58$ $\quad$ $14 \times 23 + 58$

   (b) What property of operations is demonstrated by the fact that two of the above expressions have the same value?

   ....

7. Consider your answers for question 6.

   (a) Does multiplication distribute over addition in the case of integers? ....

   (b) Illustrate your answer with two examples.

   ....

   ....

8. Three numerical expressions are given below. Underline the expressions that you would expect to have the same answers. Do not do the calculations.

   $10 \times \mathbf{(}(-50) - (-30)\mathbf{)}$ $\quad$ $10 \times (-50) - (-30)$ $\quad$ $10 \times (-50) - 10 \times (-30)$

9. Do the three sets of calculations given in question 8.

   ....

Your work in questions 5, 8 and 9 demonstrates that multiplication with a positive number distributes over addition and subtraction of integers. For example:

$10 \times \mathbf{(}5 + (-3)\mathbf{)} = 10 \times 2 = \mathbf{20}$ and $10 \times 5 + 10 \times (-3) = 50 + (-30) = \mathbf{20}$

$10 \times \mathbf{(}5 - (-3)\mathbf{)} = 10 \times 8 = \mathbf{80}$ and $10 \times 5 - 10 \times (-3) = 50 - (-30) = \mathbf{80}$

10. Calculate: $(-10) \times \mathbf{(}5 + (-3)\mathbf{)}$ ....

Now consider the question of whether multiplication with a negative number distributes over addition and subtraction of integers. For example, would $(-10) \times 5 + (-10) \times (-3)$ also have the answer -20, as does $(-10) \times \mathbf{(}5 + (-3)\mathbf{)}$?

11. What must $(-10) \times (-3)$ be equal to, if we want $(-10) \times 5 + (-10) \times (-3)$ to be equal to -20?

 ....

In order to ensure that multiplication distributes over addition and subtraction in the system of integers, we have to agree that

(a negative number) × (a negative number) is a positive number,

for example $(-10) \times (-3) = 30$.

12. Calculate.

(a) $(-10) \times (-5)$ ....

(b) $(-10) \times 5$ ....

(c) 10×5 ....

(d) $10 \times (-5)$ ....

(e) $(-20) \times (-10) + (-20) \times (-6)$

 ....

(f) $(-20) \times \mathbf{(}(-10) + (-6)\mathbf{)}$

 ....

(g) $(-20) \times (-10) - (-20) \times (-6)$

 ....

(h) $(-20) \times \mathbf{(}(-10) - (-6)\mathbf{)}$

 ....

Here is a **summary of the properties of integers** that make it possible to do calculations with integers:

- When a number is added to its additive inverse, the result is 0, for example $(+12) + (-12) = 0$.
- Adding an integer has the same effect as subtracting its additive inverse. For example, $3 + (-10)$ can be calculated by doing $3 - 10$, and the answer is -7.
- Subtracting an integer has the same effect as adding its additive inverse. For example, $3 - (-10)$ can be calculated by doing $3 + 10$, and the answer is 13.
- The product of a positive and a negative integer is negative, for example $(-15) \times 6 = -90$.
- The product of a negative and a negative integer is positive, for example $(-15) \times (-6) = 90$.

DIVISION WITH INTEGERS

1. (a) Calculate 25×8. ....

   (b) How much is $200 \div 25$? .... (c) How much is $200 \div 8$? ....

Division is the inverse of multiplication. Hence, if two numbers and the value of their product are known, the answers to two division problems are also known.

2. Calculate.

   (a) $25 \times (-8)$ (b) $(-125) \times 8$

   ....

3. Use the work you have done for question 2 to write the answers for the following division questions:

   (a) $(-1\,000) \div (-125)$ .... (b) $(-1\,000) \div 8$ ....

   (c) $(-200) \div 25$ .... (d) $(-200) \div 8$ ....

4. Can you also work out the answers for the following division questions by using the work you have done for question 2?

   (a) $1\,000 \div (-125)$ .... (b) $(-1\,000) \div (-8)$ ....

   (c) $(-100) \div (-25)$ .... (d) $100 \div (-25)$ ....

When two numbers are multiplied, for example $30 \times 4 = 120$, the word "product" can be used in various ways to describe the situation:

- An expression that specifies multiplication only, such as 30×4, is called a **product** or a product expression.
- The answer obtained is also called the product of the two numbers. For example, 120 is called the **product of 30 and 4**.

An expression that specifies division only, such as $30 \div 5$, is called a **quotient** or a **quotient expression**. The answer obtained is also called the quotient of the two numbers. For example, 6 is called the **quotient of 30 and 5.**

5. In each case, state whether you agree or disagree with the statement, and give an example to illustrate your answer.

   (a) The quotient of a positive and a negative integer is negative.

   ....

(b) The quotient of a positive and a positive integer is negative.

....

(c) The quotient of a negative and a negative integer is negative.

....

(d) The quotient of a negative and a negative integer is positive.

....

6. Do the necessary calculations to enable you to provide the values of the quotients.

(a) (–500) ÷ (–20) ....

(b) (–144) ÷ 6 ....

(c) 1 440 ÷ (–60) ....

(d) (–1 440) ÷ (–6) ....

(e) –14 400 ÷ 600 ....

(f) 500 ÷ (–20) ....

THE ASSOCIATIVE PROPERTIES OF OPERATIONS WITH INTEGERS

Multiplication of whole numbers is **associative**. This means that in a product with several factors, the factors can be placed in any sequence, and the calculations can be performed in any sequence. For example, the following sequences of calculations will all produce the same answer:

A. 2 × 3, the answer of 2 × 3 multiplied by 5, the new answer multiplied by 10
B. 2 × 5, the answer of 2 × 5 multiplied by 10, the new answer multiplied by 3
C. 10 × 5, the answer of 10 × 5 multiplied by 3, the new answer multiplied by 2
D. 3 × 5, the answer of 3 × 5 multiplied by 2, the new answer multiplied by 10

1. Do the four sets of calculations given in A to D to check whether they really produce the same answers.

A. ....

B. ....

C. ....

D. ....

2. (a) If the numbers 3 and 10 in the calculation sequences A, B, C and D are replaced with –3 and –10, do you think the four answers will still be the same? ....

   (b) Investigate, to check your expectation.

....

....

....

....

Multiplication with integers is associative.

The calculation sequence A can be represented in symbols in only two ways:

- 2 × 3 × 5 × 10. The convention to work from left to right unless otherwise indicated with brackets ensures that this representation corresponds to A.
- 5 × (2 × 3) × 10, where brackets are used to indicate that 2 × 3 should be calculated first. When brackets are used, there are different possibilities to describe the same sequence.

3. Express the calculation sequences B, C and D given on page 44 symbolically, without using brackets.

....

....

....

4. Investigate, in the same way that you did for multiplication in question 2, whether addition with integers is associative. Use sequences of four integers.

....

....

....

....

5. (a) Calculate: 80 – 30 + 40 – 20 ....

   (b) Calculate: 80 + (–30) + 40 + (–20) ....

   (c) Calculate: 30 – 80 + 20 – 40 ....

   (d) Calculate: (–30) + 80 + (– 20) + 40 ....

   (e) Calculate: 20 + 30 – 40 – 80 ....

MIXED CALCULATIONS WITH INTEGERS

1. Calculate.

(a) $-3 \times 4 + (-7) \times 9$

....

....

(b) $-20(-4 - 7)$

....

....

(c) $20 \times (-5) - 30 \times 7$

....

....

(d) $-9(20 - 15)$

....

....

(e) $-8 \times (-6) - 8 \times 3$

....

....

(f) $(-26 - 13) \div (-3)$

....

....

(g) $-15 \times (-2) + (-15) \div (-3)$

....

....

(h) $-15(2 - 3)$

....

....

(i) $(-5 + -3) \times 7$

....

....

(j) $-5 \times (-3 + 7) + 20 \div (-4)$

....

....

2. Calculate.

(a) $20 \times (-15 + 6) - 5 \times (-2 - 8) - 3 \times (-3 - 8)$

....

....

(b) $40 \times (7 + 12 - 9) + 25 \div (-5) - 5 \div 5$

....

....

(c) $-50(20 - 25) + 30(-10 + 7) - 20(-16 + 12)$

....

....

....

(d) $-5 \times (-3 + 12 - 9)$

....

....

(e) $-4 \times (30 - 50) + 7 \times (40 - 70) - 10 \times (60 - 100)$

....

....

(f) $-3 \times (-14 + 6) \times (-13 + 7) \times (-20 + 5)$

....

....

(g) $20 \times (-5) + 10 \times (-3) + (-5) \times (-6) - (3 \times 5)$

....

....

(h) $-5(-20 - 5) + 10(-7 - 3) - 20(-15 - 5) + 30(-40 - 35)$

....

....

(i) $(-50 + 15 - 75) \div (-11) + (6 - 30 + 12) \div (-6)$

....

....

2.4 Squares, cubes and roots with integers

SQUARES AND CUBES OF INTEGERS

1. Calculate.

(a) 20×20 ....

(b) $20 \times (-20)$ ....

2. Write the answers for each of the following:

(a) $(-20) \times 20$ ....

(b) $(-20) \times (-20)$ ....

3. Complete the table.

x	1	–1	2	–2	5	–5	10	–10
x^2 which is $x \times x$								
x^3								

4. In each case, state for which values of x, in the table in question 3, the given statement is true.
   (a) x^3 is a negative number ....
   (b) x^2 is a negative number ....
   (c) $x^2 > x^3$ ....
   (d) $x^2 < x^3$ ....

5. Complete the table.

x	3	–3	4	–4	6	–6	7	–7
x^2								
x^3								

6. Ben thinks of a number. He adds 5 to it, and his answer is 12.
   (a) What number did he think of? ....
   (b) Is there another number that would also give 12 when 5 is added to it? ....

7. Lebo also thinks of a number. She multiplies the number by itself and gets 25.
   (a) What number did she think of? ....
   (b) Is there more than one number that will give 25 when multiplied by itself? ......

8. Mary thinks of a number and calculates (the number) × (the number) × (the number). Her answer is 27.
   What number did Mary think of? .....

10^2 is 100 and $(–10)^2$ is also 100.
Both 10 and (–10) are called **square roots** of 100.
10 may be called the **positive square root** of 100,
and (–10) may be called the **negative square root**
of 100.

9. Write the positive square root and the negative square root of each number.

(a) 64 ....   (b) 9 ....

10. Complete the table.

Number	1	4	9	16	25	36	49	64
Positive square root			3					8
Negative square root			–3					–8

11. Complete the tables.

(a)

x	1	2	3	4	5	6	7	8
x^3								

(b)

x	–1	–2	–3	–4	–5	–6	–7	–8
x^3								

3^3 is 27 and $(-5)^3$ is –125.
3 is called the **cube root** of 27, because $3^3 = 27$.
–5 is called the cube root of –125 because $(-5)^3 = -125$.

12. Complete the table.

Number	–1	8	–27	–64	–125	–216	1 000
Cube root			–3				10

The symbol $\sqrt{\ }$ is used to indicate "root".

$\sqrt[3]{-125}$ represents the cube root of –125. That means $\sqrt[3]{-125} = -5$.

$\sqrt[2]{36}$ represents the positive square root of 36, and $-\sqrt[2]{36}$ represents the negative square root. The "2" that indicates "square" is normally omitted, so $\sqrt{36} = 6$ and $-\sqrt{36} = -6$.

13. Complete the table.

$\sqrt[3]{-8}$	$\sqrt{121}$	$\sqrt[3]{-64}$	$-\sqrt{64}$	$\sqrt{64}$	$\sqrt[3]{-1}$	$-\sqrt{1}$	$\sqrt[3]{-216}$

WORKSHEET

1. Use the numbers –8, –5 and –3 to demonstrate each of the following:

   (a) Multiplication with integers distributes over addition.

   ....

   (b) Multiplication with integers distributes over subtraction.

   ....

   (c) Multiplication with integers is associative.

   ....

   (d) Addition with integers is associative.

   ....

2. Calculate each of the following without using a calculator:

   (a) $5 \times (-2)^3$ (b) $3 \times (-5)^2$

   ....

   (c) $2 \times (-5)^3$ (d) $10 \times (-3)^2$

   ....

3. Use a calculator to calculate each of the following:

   (a) $24 \times (-53) + (-27) \times (-34) - (-55) \times 76$

   ....

   (b) $64 \times (27 - 85) - 29 \times (-47 + 12)$

   ....

4. Use a calculator to calculate each of the following:

   (a) $-24 \times 53 + 27 \times 34 + 55 \times 76$ ....

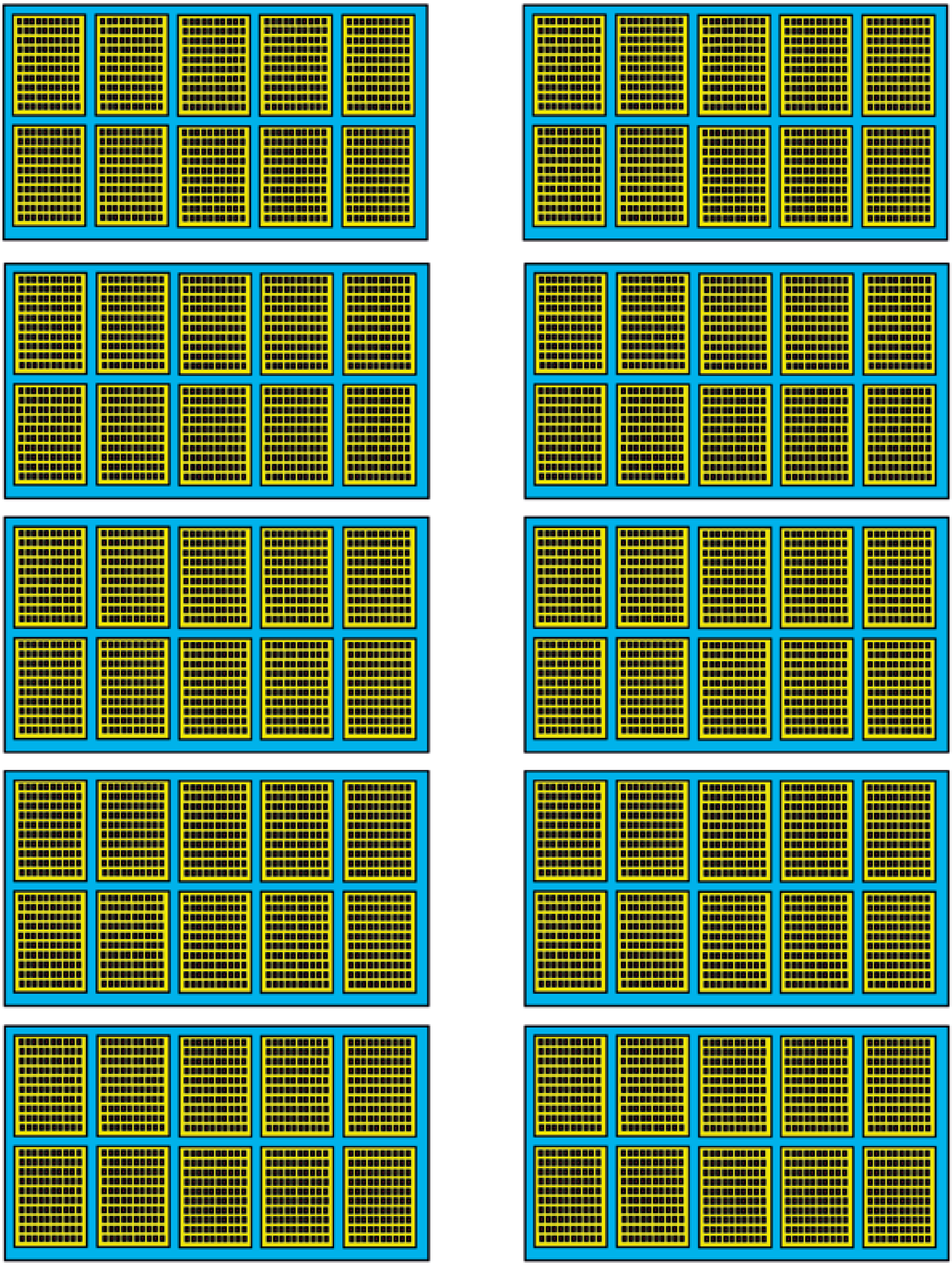
   (b) $64 \times (-58) + 29 \times (47 - 12)$ ....

If you don't get the same answers in questions 3 and 4, you have made mistakes.

Chapter 3
Exponents

In this chapter, you will revise work you have done on squares, cubes, square roots and cube roots. You will learn about laws of exponents that will enable you to do calculations using numbers written in exponential form.

Very large numbers are written in scientific notation. Scientific notation is a convenient way of writing very large numbers as a product of a number between 1 and 10 and a power of 10.



3 Exponents

3.1 Revision

EXPONENTIAL NOTATION

1. Calculate.

(a) $2 \times 2 \times 2$

....

(b) $2 \times 2 \times 2 \times 2 \times 2 \times 2$

....

(c) $3 \times 3 \times 3$

....

(d) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

....

Instead of writing $3 \times 3 \times 3 \times 3 \times 3 \times 3$ we can write 3^6.
We read this as "3 to the power of 6". The number 3 is the **base**, and 6 is the **exponent**.
When we write $3 \times 3 \times 3 \times 3 \times 3 \times 3$ as 3^6, we are using **exponential notation**.

2. Write each of the following in exponential form:

(a) $2 \times 2 \times 2$

....

(b) $2 \times 2 \times 2 \times 2 \times 2 \times 2$

....

(c) $3 \times 3 \times 3$

....

(d) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

....

3. Calculate.

(a) 5^2

....

(b) 2^5

....

(c) 10^2

....

(d) 15^2

....

(e) 3^4

....

(f) 4^3

....

(g) 2^3

....

(h) 3^2

....

SQUARES

To square a number is to multiply it by itself. The square of 8 is 64 because 8×8 equals 64.
We write 8×8 as 8^2 in exponential form.
We read 8^2 as **eight squared**.

1. Complete the table.

	Number	Square the number	Exponential form	Square
(a)	1			
(b)	2			
(c)	3			
(d)	4			
(e)	5			
(f)	6			
(g)	7			
(h)	8	8×8	8^2	64
(i)	9			
(j)	10			
(k)	11			
(l)	12			

2. Calculate the following:

(a) $3^2 \times 4^2$

....

(b) $2^2 \times 3^2$

....

(c) $2^2 \times 5^2$

....

(d) $2^2 \times 4^2$

....

3. Complete the following statements to make them true:

(a) $3^2 \times 4^2 = \ldots\ldots^2$

(b) $2^2 \times 3^2 = \ldots\ldots^2$

(c) $2^2 \times 5^2 = \ldots\ldots^2$

(d) $2^2 \times 4^2 = \ldots\ldots^2$

CUBES

To cube a number is to multiply it by itself and then by itself again. The cube of 3 is 27 because $3 \times 3 \times 3$ equals 27.

We write $3 \times 3 \times 3$ as 3^3 in exponential form.

We read 3^3 as **three cubed**.

1. Complete the table.

	Number	Cube the number	Exponential form	Cube
(a)	1			
(b)	2			
(c)	3	$3 \times 3 \times 3$	3^3	27
(d)	4			
(e)	5			
(f)	6			
(g)	7			
(h)	8			
(i)	9			
(j)	10			

2. Calculate the following:

(a) $2^3 \times 3^3$

....

(b) $2^3 \times 5^3$

....

(c) $2^3 \times 4^3$

....

(d) $1^3 \times 9^3$

....

3. Which of the following statements are true? If a statement is false, rewrite it as a true statement.

(a) $2^3 \times 3^3 = 6^3$

....

(b) $2^3 \times 5^3 = 7^3$

....

(c) $2^3 \times 4^3 = 8^3$

....

(d) $1^3 \times 9^3 = 10^3$

....

SQUARE AND CUBE ROOTS

To find the square root of a number we ask the question: Which number was multiplied by itself to get a square?

The square root of 16 is 4 because $4 \times 4 = 16$.

The question: **Which number was multiplied by itself to get 16?** is written mathematically as $\sqrt{16}$. The answer to this question is written as $\sqrt{16} = 4$.

1. Complete the table.

	Number	Square of the number	Square root of the square of the number	Reason
(a)	1			
(b)	2			
(c)	3			
(d)	4	16	4	$4 \times 4 = 16$
(e)	5			
(f)	6			
(g)	7			
(h)	8			
(i)	9			
(j)	10			
(k)	11			
(l)	12			

2. Calculate the following. Justify your answer.

(a) $\sqrt{144}$

....

(b) $\sqrt{100}$

....

(c) $\sqrt{81}$

....

(d) $\sqrt{64}$

....

To find the cube root of a number we ask the question: Which number was multiplied by itself and again by itself to get a cube?

The cube root of 64 is 4 because $4 \times 4 \times 4 = 64$.

The question: **Which number was multiplied by itself and again by itself (or cubed) to get 64?** is written mathematically as $\sqrt[3]{64}$. The answer to this question is written as $\sqrt[3]{64} = 4$.

3. Complete the table.

	Number	Cube of the number	Cube root of the cube of the number	Reason
(a)	1			
(b)	2			
(c)	3			
(d)	4	64	4	$4 \times 4 \times 4 = 64$
(e)	5			
(f)	6			
(g)	7			
(h)	8			
(i)	9			
(j)	10			

4. Calculate the following and give reasons for your answers:

(a) $\sqrt[3]{216}$

....

(b) $\sqrt[3]{8}$

....

(c) $\sqrt[3]{125}$

....

(d) $\sqrt[3]{27}$

....

(e) $\sqrt[3]{64}$

....

(f) $\sqrt[3]{1\ 000}$

....

3.2 Working with integers

REPRESENTING INTEGERS IN EXPONENTIAL FORM

1. Calculate the following, without using a calculator:

(a) $-2 \times -2 \times -2$

....

(b) $-2 \times -2 \times -2 \times -2$

....

(c) -5×-5

....

(d) $-5 \times -5 \times -5$

....

(e) $-1 \times -1 \times -1 \times -1$

....

(f) $-1 \times -1 \times -1$

....

2. Calculate the following:

(a) -2^2

....

(b) $(-2)^2$

....

(c) $(-5)^3$

....

(d) -5^3

....

3. Use your calculator to calculate the answers to question 2.

(a) Are your answers to question 2(a) and (b) different or the same as those of the calculator?

....

(b) If your answers are different to those of the calculator, try to explain how the calculator did the calculations differently from you.

....

....

....

....

The calculator "understands" $\mathbf{-5^2}$ and $\mathbf{(-5)^2}$ as two different numbers.

It understands $\mathbf{-5^2}$ as $\mathbf{-5 \times 5 = -25}$ and $\mathbf{(-5)^2}$ as $\mathbf{-5 \times -5 = 25}$

4. Write the following in exponential form:
   (a) $-2 \times -2 \times -2$ (b) $-2 \times -2 \times -2 \times -2$

   ....

   (c) -5×-5 (d) $-5 \times -5 \times -5$

   ....

   (e) $-1 \times -1 \times -1 \times -1$ (f) $-1 \times -1 \times -1$

   ....

5. Calculate the following:
   (a) $(-3)^2$ (b) $(-3)^3$

   ....

   ....

   (c) $(-2)^4$ (d) $(-2)^6$

   ....

   ....

   (e) $(-2)^5$ (f) $(-3)^4$

   ....

   ....

6. Say whether the sign of the answer is negative or positive. Explain why.
   (a) $(-3)^6$ (b) $(-5)^{11}$

   ....

   (c) $(-4)^{20}$ (d) $(-7)^5$

   ....

7. Say whether the following statements are true or false. If a statement is false rewrite it as a correct statement.
   (a) $(-3)^2 = -9$ (b) $-3^2 = 9$

   ....

   (c) $(-5^2) = -5^2$ (d) $(-1)^3 = -1^3$

   ....

   (e) $(-6)^3 = -18$ (f) $(-2)^6 = 2^6$

   ....

3.3 Laws of exponents

PRODUCT OF POWERS

1. A product of 2s is given below. Describe it using exponential notation, that is, write it as a power of 2.

   $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

   ....

2. Express each of the following as a product of the powers of 2, as indicated by the brackets.

   (a) $(2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$

   ....

   (b) $(2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2)$

   ....

   (c) $(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$

   ....

   (d) $(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

   ....

   (e) $(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2)$

   ....

3. Complete the following statements so that they are true. You may want to refer to your answers to question 2 (a) to (e) to help you.

   (a) $2^3 \times$ ...... $= 2^{12}$

   (b) $2^5 \times$ ...... $\times 2^2 = 2^{12}$

   (c) $2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 =$ ......

   (d) $2^8 \times$ ...... $= 2^{12}$

   (e) $2^3 \times 2^3 \times 2^3 \times$ ...... $= 2^{12}$

   (f) $2^6 \times$ ...... $= 2^{12}$

   (g) $2^2 \times 2^{10} =$ ......

Suppose we are asked to simplify: $3^2 \times 3^4$.

The solution is: $3^2 \times 3^4 = 9 \times 81$

$= 729$

$= 3^6$

The base (3) is a repeated factor. The exponents (2 and 4) tell us the number of times each factor is repeated.

We can explain this solution in the following manner:

$$3^2 \times 3^4 = \underbrace{3 \times 3}_{\text{2 factors}} \times \underbrace{3 \times 3 \times 3 \times 3}_{\text{4 factors}} = \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3}_{\text{6 factors}} = 3^6$$

4. Complete the table.

	Product of powers	Repeated factor	Total number of times the factor is repeated	Simplified form
(a)	$2^7 \times 2^3$			
(b)	$5^2 \times 5^4$			
(c)	$4^1 \times 4^5$			
(d)	$6^3 \times 6^2$			
(e)	$2^8 \times 2^2$			
(f)	$5^3 \times 5^3$			
(g)	$4^2 \times 4^4$			
(h)	$2^1 \times 2^9$			

When you multiply two or more powers that have the same base, the answer has the same base, but its exponent is equal to the sum of the exponents of the numbers you are multiplying.

We can express this symbolically as $a^m \times a^n = a^{m+n}$, where m and n are natural numbers and a is not zero.

5. What is wrong with these statements? Correct each one.

(a) $2^3 \times 2^4 = 2^{12}$

....

(b) $10 \times 10^2 \times 10^3 = 10^{1 \times 2 \times 3} = 10^6$

....

(c) $3^2 \times 3^3 = 3^6$

....

(d) $5^3 \times 5^2 = 15 \times 10$

....

6. Express each of the following numbers as a single power of 10.

   Example: 1 000 000 as a power of 10 is 10^6.

   (a) 100 (b) 1 000 (c) 10 000

   ....

   (d) $10^2 \times 10^3 \times 10^4$ (e) $100 \times 1\,000 \times 10\,000$ (f) 1 000 000 000

   ....

   ....

7. Write each of the following products in exponential form:

   (a) $x \times x \times x \times x \times x \times x \times x \times x \times x =$ ......

   (b) $(x \times x) \times (x \times x \times x) \times (x \times x \times x \times x)$

   ....

   (c) $(x \times x \times x \times x) \times (x \times x) \times (x \times x) \times x$

   ....

   (d) $(x \times x \times x \times x \times x \times x) \times (x \times x \times x)$

   ....

   (e) $(x \times x \times x) \times (y \times y \times y)$

   ....

   (f) $(a \times a) \times (b \times b)$

   ....

8. Complete the table.

	Product of powers	Repeated factor	Total number of times the factor is repeated	Simplified form
(a)	$x^7 \times x^3$			
(b)	$x^2 \times x^4$			
(c)	$x^1 \times x^5$			
(d)	$x^3 \times x^2$			
(e)	$x^8 \times x^2$			
(f)	$x^3 \times x^3$			
(g)	$x^1 \times x^9$			

RAISING A POWER TO A POWER

1. Complete the table of powers of 2.

x	1	2	3	4	5	6	7	8	9	10	11
2^x	2	4									
	2^1	2^2	2^3								

x	12	13	14	15	16	17	18
2^x							

2. Complete the table of powers of 3.

x	1	2	3	4	5	6	7	8	9
3^x									
	3^1	3^2	3^3						

x	10	11	12	13	14
3^x					

3. Complete the table. You can read the values from the tables you made in questions 1 and 2.

Product of powers	**Repeated factor**	**Power of power notation**	**Total number of repetitions**	**Simplified form**	**Value**
$2^4 \times 2^4 \times 2^4$	2	$(2^4)^3$	12	2^{12}	4 096
$3^2 \times 3^2 \times 3^2 \times 3^2$					
$2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3$					
$3^4 \times 3^4 \times 3^4$					
$2^6 \times 2^6 \times 2^6$					

4. Use your table of powers of 2 to find the answers for the following:

(a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ = ... = ...

(b) $(2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$ = ... = ...

(c) 16^3 = ... = ... = ...

5. Use your table of powers of 2 to find the answers for the following:

(a) Is $16^3 = 2^{12}$? ......
(b) Is $2^4 \times 2^4 \times 2^4 = 2^{12}$ ......
(c) Is $2^4 \times 2^3 = 2^{12}$? ......
(d) Is $(2^4)^3 = 2^4 \times 2^4 \times 2^4$? ......
(e) Is $(2^4)^3 = 2^{12}$? ......
(f) Is $(2^4)^3 = 2^{4+3}$? ......
(g) Is $(2^4)^3 = 2^{4 \times 3}$? ......
(h) Is $(2^2)^5 = 2^{2+5}$? ......

6. (a) Express 8^5 as a power of 2. It may help to first express 8 as a power of 2.

....

(b) Can $(2^3) \times (2^3) \times (2^3) \times (2^3) \times (2^3)$ be expressed as $(2^3)^5$? ....

(c) Is $(2^3)^5 = 2^{3+5}$ or is $(2^3)^5 = 2^{3 \times 5}$? ....

7. (a) Express 4^3 as a power of 2.

....

(b) Calculate $2^2 \times 2^2 \times 2^2$ and express your answer as a single power of 2.

....

(c) Can $(2^2) \times (2^2) \times (2^2)$ be expressed as $(2^2)^3$? ....

(d) Is $(2^2)^3 = 2^{2+3}$ or is $(2^2)^3 = 2^{2 \times 3}$? ....

8. Simplify the following.

Example: $(10^2)^2 = 10^2 \times 10^2 = 10^{2+2} = 10^4 = 10\,000$

(a) $(3^3)^2$ ....
(b) $(4^3)^2$ ....
(c) $(2^4)^2$ ....
(d) $(9^2)^2$ ....
(e) $(3^3)^3$ ....
(f) $(4^3)^3$ ....
(g) $(5^4)^3$ ....
(h) $(9^2)^3$ ....

$(a^m)^n = a^{m \times n}$, where m and n are natural numbers and a is not equal to zero.

9. Simplify.

(a) $(5^4)^{10}$ ....
(b) $(10^4)^5$ ....
(c) $(6^4)^4$ ....
(d) $(5^4)^{10}$ ....

10. Write 5^{12} as a power of powers of 5 in two different ways. ....

To simplify $(x^2)^5$ we can write it out as a product of powers or we can use a shortcut.

$$(x^2)^5 = x^2 \times x^2 \times x^2 \times x^2 \times x^2$$

$$= \underbrace{x \times x}_{} \times \underbrace{x \times x}_{} \times \underbrace{x \times x}_{} \times \underbrace{x \times x}_{} \times \underbrace{x \times x}_{} = x^{10}$$

2×5 factors = 10 factors

11. Complete the table.

	Expression	Write as a product of the powers and then simplify	Use the rule $(a^m)^n$ to simplify
(a)	$(a^4)^5$	$a^4 \times a^4 \times a^4 \times a^4 \times a^4$ $= a^{4+4+4+4+4} = a^{20}$	$(a^4)^5 = a^{4 \times 5} = a^{20}$
(b)	$(b^{10})^5$		
(c)	$(x^7)^3$		
(d)		$s^6 \times s^6 \times s^6 \times s^6$ $= s^{6+6+6+6}$ $= s^{24}$	
(e)			$y^{3 \times 7} = y^{21}$

POWER OF A PRODUCT

1. Complete the table. You may use your calculator when you are not sure of a value.

	x	1	2	3	4	5
(a)	2^x	$2^1 = 2$				
(b)	3^x		$3^2 = 9$			
(c)	6^x			$6^3 = 216$		

2. Use the table in question 1 to answer the questions below. Are these statements true or false? If a statement is false rewrite it as a correct statement.

(a) $6^2 = 2^2 \times 3^2$

....

(b) $6^3 = 2^3 \times 3^3$

....

(c) $6^5 = 2^5 \times 3^5$

....

(d) $6^8 = 2^4 \times 3^4$

....

3. Complete the table.

	Expression	The bases of the expression are factors of ...	Equivalent expression
(a)	$2^6 \times 5^6$	10	10^6
(b)	$3^2 \times 4^2$		
(c)	$4^2 \times 2^2$		
(d)			56^5
(e)			30^3
(f)	$3^5 \times x^5$	$3x$	$(3x)^5$
(g)	$7^2 \times z^2$		
(h)	$4^3 \times y^3$		
(i)			$(2m)^6$
(j)			$(2m)^3$
(k)	$2^{10} \times y^{10}$		$(2y)^{10}$

12^2 can be written in terms of its factors as $(2 \times 6)^2$ or as $(3 \times 4)^2$.
We already know that $12^2 = 144$.
What this tells us is that both $(2 \times 6)^2$ and $(3 \times 4)^2$ also equal 144.

We write

$$\begin{aligned} 12^2 &= (2 \times 6)^2 \\ &= 2^2 \times 6^2 \\ &= 4 \times 36 \\ &= 144 \end{aligned}$$

or

$$\begin{aligned} 12^2 &= (3 \times 4)^2 \\ &= 3^2 \times 4^2 \\ &= 9 \times 16 \\ &= 144 \end{aligned}$$

A product raised to a power is the product of the factors each raised to the same power.

Using symbols, we write $(a \times b)^m = a^m \times b^m$, where m is a natural number and a and b are not equal to zero.

4. Write each of the following expressions as an expression with one base:

Example: $3^{10} \times 2^{10} = (3 \times 2)^{10} = 6^{10}$

(a) $3^2 \times 5^2$ (b) $5^3 \times 2^3$ (c) $7^4 \times 4^4$

(d) $2^3 \times 6^3$ (e) $4^4 \times 2^4$ (f) $5^2 \times 7^2$

5. Write the following as a product of powers:

Example: $(3x)^3 = 3^3 \times x^3 = 27x^3$

(a) 6^3 (b) 15^2 (c) 21^4

(d) 6^5 (e) 18^2 (f) $(st)^7$

(g) $(ab)^3$ (h) $(2x)^2$ (i) $(3y)^5$

(j) $(3c)^2$ (k) $(gh)^4$ (l) $(4x)^3$

6. Simplify the following expressions:

Example: $3^2 \times m^2 = 9 \times m^2 = 9m^2$

(a) $3^5 \times b^5$ (b) $2^6 \times y^6$ (c) $x^2 \times y^2$

(d) $10^4 \times x^4$ (e) $3^3 \times x^3$ (f) $5^2 \times t^2$

(g) $6^3 \times m^7$ (h) $12^2 \times a^2$ (i) $n^3 \times p^9$

A QUOTIENT OF POWERS

Consider the following table:

x	1	2	3	4	5	6
2^x	2	4	8	16	32	64
3^x	3	9	27	81	243	729
5^x	5	25	125	625	3 125	15 625

Answer questions 1 to 4 by referring to the table when you need to.

1. Give the value of each of the following:

   (a) 3^4 (b) 2^5 (c) 5^6

   ....

2. (a) Calculate $3^6 \div 3^3$ (Read the values of 3^6 and 3^3 from the table and then divide. You may use a calculator where necessary.)

   ....

   (b) Calculate 3^{6-3}

   ....

   (c) Is $3^6 \div 3^3$ equal to 3^3? Explain.

   ....

> To calculate 4^{5-3} we first do the calculation in the exponent, that is, we subtract 3 from 5. Then we can calculate 4^2 as $4 \times 4 = 16$.

3. (a) Calculate the value of 2^{6-2}

   ....

   (b) Calculate the value of $2^6 \div 2^2$

   ....

   (c) Calculate the value of $2^{6 \div 2}$

   ....

   (d) Read from the table the value of 2^3

   ....

   (e) Read from the table the value of 2^4

   ....

   (f) Which of the statements below is true? Give an explanation for your answer.

   A. $2^6 \div 2^2 = 2^{6-2} = 2^4$ B. $2^6 \div 2^2 = 2^{6 \div 2} = 2^3$

   ....

   ....

4. Say which of the statements below are true and which are false. If a statement is false rewrite it as a correct statement.

(a) $5^6 \div 5^4 = 5^{6 \div 4}$

(b) $3^{4-1} = 3^4 \div 3$

(c) $5^6 \div 5 = 5^{6-1}$

(d) $2^5 \div 2^3 = 2^2$

$a^m \div a^n = a^{m-n}$
where m and n are natural numbers and m is a number greater than n and a is not zero.

5. Simplify the following. Do not use a calculator.

Example: $3^{17} \div 3^{12} = 3^{17-12} = 3^5 = 243$

(a) $2^{12} \div 2^{10}$

(b) $6^{17} \div 6^{14}$

(c) $10^{20} \div 10^{14}$

(d) $5^{11} \div 5^8$

6. Simplify:

(a) $x^{12} \div x^{10}$

(b) $y^{17} \div y^{14}$

(c) $t^{20} \div t^{14}$

(d) $n^{11} \div n^8$

THE POWER OF ZERO

1. Simplify the following:

(a) $2^{12} \div 2^{12}$

....

....

(b) $6^{17} \div 6^{17}$

....

....

(c) $6^{14} \div 6^{14}$

....

....

(d) $2^{10} \div 2^{10}$

....

....

We define $a^0 = 1$.
Any number raised to the power of zero is always equal to 1.

2. Simplify the following:

(a) 100^0 ....

(b) x^0 ....

(c) $(100x)^0$ ....

(d) $(5x^3)^0$ ....

3.4 Calculations

MIXED OPERATIONS

Simplify the following:

1. $3^3 + \sqrt[3]{-27} \times 2$

....

....

....

2. $5 \times (2 + 3)^2 + (-1)^0$

....

....

....

3. $3^2 \times 2^3 + 5 \times \sqrt{100}$

....

....

....

4. $\dfrac{\sqrt[3]{1\,000}}{\sqrt{100}} + (4 - 1)^2$

....

....

....

5. $\sqrt{16} \times \sqrt{16} + \sqrt[3]{216} + 3^2 \times 10$

....

....

....

6. $4^3 \div 2^3 + \sqrt{144}$

....

....

....

3.5 Squares, cubes and roots of rational numbers

SQUARING A FRACTION

Squaring or cubing a fraction or a decimal fraction is no different from squaring or cubing an integer.

1. Complete the table.

	Fraction	Square the fraction	Value of the square of the fraction
(a)	$\frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
(b)	$\frac{2}{3}$		
(c)	$\frac{3}{4}$		
(d)	$\frac{2}{5}$		
(e)	$\frac{3}{5}$		
(f)	$\frac{2}{6}$		
(g)	$\frac{3}{7}$		
(h)	$\frac{11}{12}$		

2. Calculate the following:

(a) $\left(\frac{3}{2}\right)^2$ (b) $\left(\frac{4}{5}\right)^2$ (c) $\left(\frac{7}{8}\right)^2$

....

3. (a) Use the fact that 0,6 can be written as $\frac{6}{10}$ to calculate $(0,6)^2$.

....

(b) Use the fact that 0,8 can be written as $\frac{8}{10}$ to calculate $(0,8)^2$.

....

FINDING THE SQUARE ROOT OF A FRACTION

1. Complete the table.

	Fraction	Writing the fraction as a product of factors	Square root
(a)	$\frac{81}{121}$		
(b)	$\frac{64}{81}$		
(c)	$\frac{49}{169}$		
(d)	$\frac{100}{225}$		

2. Determine the following:

(a) $\sqrt{\frac{25}{16}}$ (b) $\sqrt{\frac{81}{144}}$ (c) $\sqrt{\frac{400}{900}}$ (d) $\sqrt{\frac{36}{81}}$

....

3. (a) Use the fact that 0,01 can be written as $\frac{1}{100}$ to calculate $\sqrt{0,01}$.

....

(b) Use the fact that 0,49 can be written as $\frac{49}{100}$ to calculate $\sqrt{0,49}$.

....

4. Calculate the following:

(a) $\sqrt{0,09}$ (b) $\sqrt{0,64}$ (c) $\sqrt{1,44}$

....

CUBING A FRACTION

One half cubed is equal to one eighth.

We write this as $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

1. Calculate the following:

(a) $\left(\frac{2}{3}\right)^3$ (b) $\left(\frac{5}{10}\right)^3$ (c) $\left(\frac{5}{6}\right)^3$ (d) $\left(\frac{4}{5}\right)^3$

....

....

2. (a) Use the fact that 0,6 can be written as $\frac{6}{10}$ to find $(0,6)^3$.

....

(b) Use the fact that 0,8 can be written as $\frac{8}{10}$ to calculate $(0,8)^3$.

....

(c) Use the fact that 0,7 can be written as $\frac{7}{10}$ to calculate $(0,7)^3$.

....

3.6 Scientific notation

VERY LARGE NUMBERS

1. Express each of the following as a single number. Do not use a calculator.
   Example: $7{,}56 \times 100$ can be written as 756.

   (a) $3{,}45 \times 100$ (b) $3{,}45 \times 10$ (c) $3{,}45 \times 1\ 000$

   ....

   (d) $2{,}34 \times 10^2$ (e) $2{,}34 \times 10$ (f) $2{,}34 \times 10^3$

   ....

   (g) $10^4 \times 10^2$ (h) $10^0 \times 10^6$ (i) $3{,}4 \times 10^5$

   ....

   ....

We can write 136 000 000 as $1{,}36 \times 10^8$.
$1{,}36 \times 10^8$ is called the **scientific notation** for 136 000 000.

In scientific notation, a number is expressed in two parts: a number between 1 and 10 multiplied by a power of 10. The exponent must always be an integer.

2. Write the following numbers in scientific notation:

   (a) 367 000 000 (b) 21 900 000

   ....

   (c) 600 000 000 000 (d) 178

   ....

3. Write each of the following numbers in the ordinary way.
   For example: $3{,}4 \times 10^5$ written in the ordinary way is 340 000.

   (a) $1{,}24 \times 10^8$ (b) $9{,}2074 \times 10^4$

   ....

   (c) $1{,}04 \times 10^6$ (d) $2{,}05 \times 10^3$

   ....

4. The age of the universe is 15 000 000 000 years. Express the age of the universe in scientific notation. ....

5. The average distance from the Earth to the Sun is 149 600 000 km. Express this distance in scientific notation. ....

Because it is easier to multiply powers of ten without a calculator, **scientific notation** makes it possible to do calculations in your head.

6. Explain why the number 24×10^3 is not in scientific notation.

....

7. Calculate the following. Do not use a calculator.

Example: $3\ 000\ 000 \times 90\ 000\ 000 = 3 \times 10^6 \times 9 \times 10^7 = 3 \times 9 \times 10^{6+7}$
$= 27 \times 10^{13} = 270\ 000\ 000\ 000\ 000$

(a) $13\ 000 \times 150\ 000$ | (b) $200 \times 6\ 000\ 000$

.... | ....

.... | ....

.... | ....

.... | ....

.... | ....

(c) $120\ 000 \times 120\ 000\ 000$ | (d) $2{,}5 \times 40\ 000\ 000$

.... | ....

.... | ....

.... | ....

.... | ....

.... | ....

8. Use > or < to compare these numbers:

(a) $1{,}3 \times 10^9$ ☐ $2{,}4 \times 10^7$ (b) $6{,}9 \times 10^2$ ☐ $4{,}5 \times 10^3$

(c) $7{,}3 \times 10^4$ ☐ $7{,}3 \times 10^2$ (d) $3{,}9 \times 10^6$ ☐ $3{,}7 \times 10^7$

WORKSHEET

1. Calculate:

   (a) 11^2 .... (b) $3^2 \times 4^2$ ....

   (c) 6^3 .... (d) $\sqrt{121}$ ....

   (e) $(-3)^2$ .... (f) $\sqrt[3]{125}$ ....

2. Simplify:

   (a) $3^4 \times m^6$ .... (b) $b^2 \times n^6$ ....

   (c) $y^{12} \div y^5$ .... (d) $(10^2)^3$ ....

   (e) $(2w^2)^3$ .... (f) $(3d^5)(2d)^3$ ....

3. Calculate:

   (a) $\left(\frac{2}{5}\right)^2$ .... (b) $\sqrt{\frac{9}{25}}$ ....

   (c) $(6^4y^2)^0$ .... (d) $(0{,}7)^2$ ....

4. Simplify:

   (a) $(2^2 + 4)^2 + \frac{6^2}{3^2}$ (b) $\sqrt[3]{-125} - 5 \times 3^2$

   ....

   ....

   ....

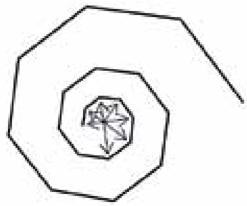
5. Write 3×10^9 in the ordinary way. ....

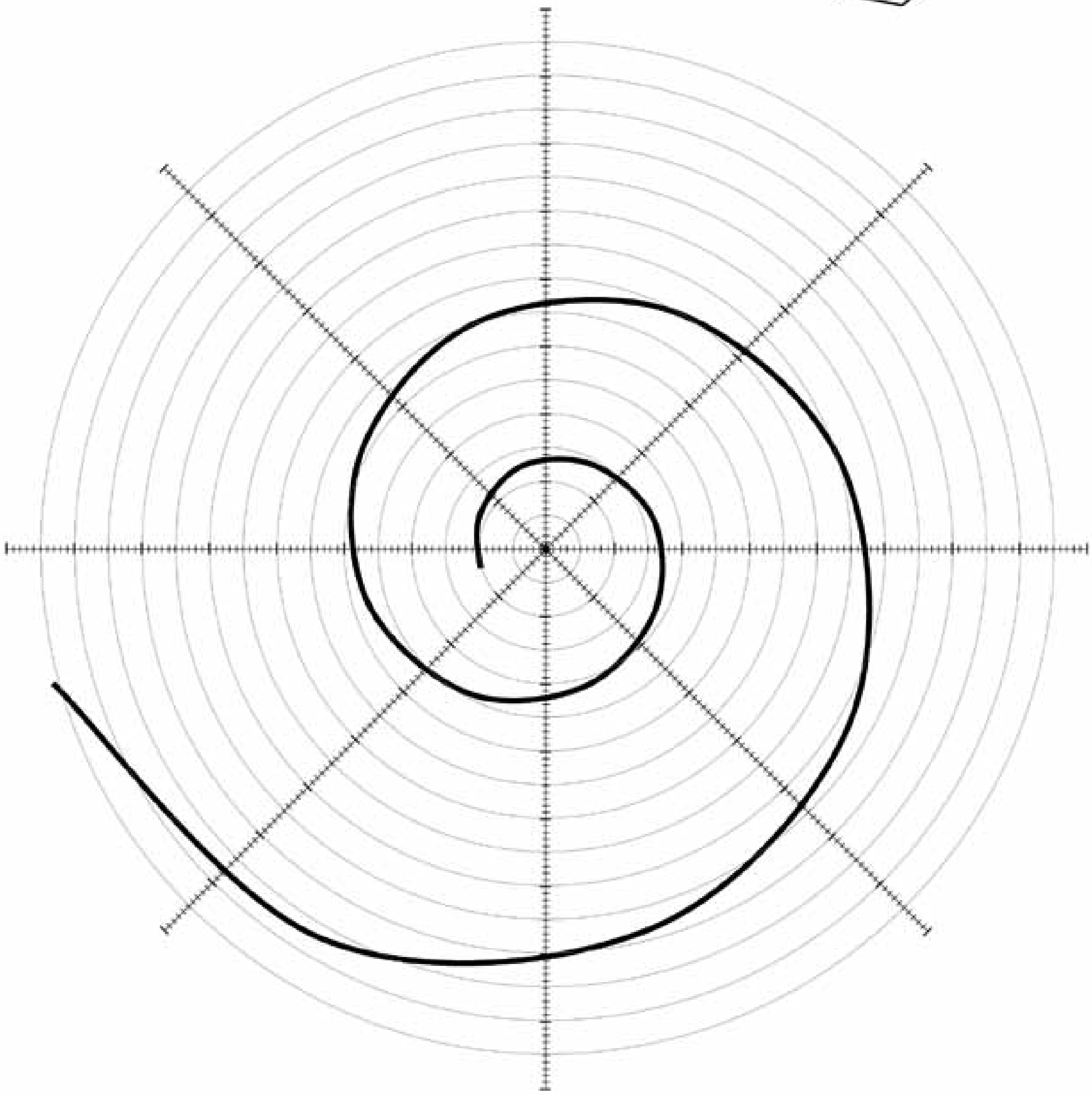
6. The first birds appeared on Earth about 208 000 000 years ago. Write this number in scientific notation.

   ....

Chapter 4
Numeric and geometric patterns

In this chapter you will learn to create, recognise, describe, extend and make generalisations about numeric and geometric patterns. Patterns allow us to make predictions. You will also work with different representations of patterns, such as flow diagrams and tables.





4 Numeric and geometric patterns

4.1 The term–term relationship in a sequence

GOING FROM ONE TERM TO THE NEXT

Write down the next three numbers in each of the sequences below. Also explain in writing, in each case, how you figured out what the numbers should be.

A list of numbers which form a pattern is called a **sequence**. Each number in a sequence is called a **term** of the sequence. The first number is the first term of the sequence.

1. Sequence A: 2; 5; 8; 11; 14; 17; 20; 23; ....

   ....

2. Sequence B: 4; 5; 8; 13; 20; 29; 40; ....

   ....

3. Sequence C: 1; 2; 4; 8; 16; 32; 64; ....

   ....

4. Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19; ....

   ....

5. Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; ....

   ....

6. Sequence F: 2; 6; 18; 54; 162; 486; ....

   ....

7. Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; ....

   ....

8. Sequence H: 2; 4; 8; 16; 32; 64; ....

   ....

Numbers that follow one another are said to be **consecutive**.

ADDING OR SUBTRACTING THE SAME NUMBER

1. Which sequences on the previous page are of the same kind as sequence A? Explain your answer.

   ....

Amanda explains how she figured out how to continue sequence A:

> *I looked at the first two numbers in the sequence and saw that I needed 3 to go from 2 to 5. I looked further and saw that I also needed 3 to go from 5 to 8. I tested that and it worked for all the next numbers.*
>
> *This gave me* ***a rule I could use to extend the sequence: add 3 to each number to find the next number in the pattern****.*

Tamara says you can also find the pattern by working backwards and subtracting 3 each time:

14 – 3 = 11; 11 – 3 = 8; 8 – 3 = 5; 5 – 3 = 2

When the **differences** between consecutive terms of a sequence are the same, we say the difference is **constant**.

2. Provide a rule to describe the relationship between the numbers in the sequence. Use this rule to calculate the missing numbers in the sequence.

   (a) 1; 8; 15; .....; .....; .....; .....; .....; ...

   ....

   (b) 10 020; ....; ....; ....; 9 980; 9 970; ....; ....; 9 940; 9 930; ...

   ....

   (c) 1,5; 3,0; 4,5; .....; .....; .....; .....; .....; ...

   ....

   (d) 2,2; 4,0; 5,8; ......; ......; ......; ......; ......; ...

   ....

   (e) $45\frac{3}{4}$; $46\frac{1}{2}$; $47\frac{1}{4}$; 48; ......; ......; ......; ......; ......; ...

   ....

   (f) ....; 100,49; 100,38; 100,27; ....; ....; 99,94; 99,83; 99,72; ...

   ....

3. Complete the table below.

Input number	1	2	3	4	5		12		n
Input number + 7	8			11		15		30	

MULTIPLYING OR DIVIDING WITH THE SAME NUMBER

Take another look at sequence F: 2; 6; 18; 54; 162; 486; ...

Piet explains that he figured out how to continue the sequence F:

I looked at the first two terms in the sequence and wrote $2 \times ? = 6$.
When I multiplied the first number by 3, I got the second number: $2 \times 3 = 6$.
I then checked to see if I could find the next number if I multiplied 6 by 3: $6 \times 3 = 18$.
I continued checking in this way: $18 \times 3 = 54$; $54 \times 3 = 162$ and so on.
This gave me ***a rule I can use to extend the sequence*** *and my rule was:* ***multiply each number by 3 to calculate the next number in the sequence.***

Zinhle says you can also find the pattern by working backwards and dividing by 3 each time:

$54 \div 3 = 18$; $18 \div 3 = 6$; $6 \div 3 = 2$

The number that we multiply with to get the next term in the sequence is called a **ratio**. If the number we multiply with remains the same throughout the sequence, we say it is a **constant ratio**.

1. Check whether Piet's reasoning works for sequence H: 2; 4; 8; 16; 32; 64; ...

   ....

2. Describe, in words, the rule for finding the next number in the sequence. Also write down the next five terms of the sequence if the pattern is continued.

   (a) 1; 10; 100; 1 000; ....

   ....

   (b) 16; 8; 4; 2; ....

   ....

   (c) 7; –21; 63; –189; ....

   ....

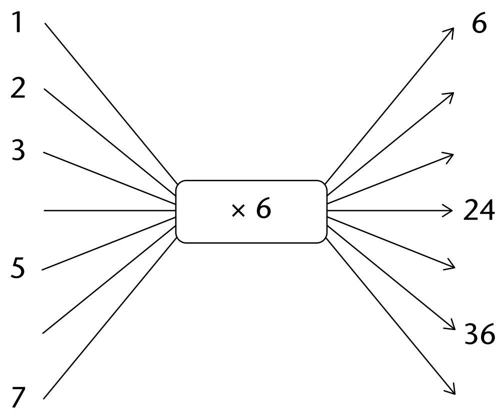
   (d) 3; 12, 48; ....

   ....

   (e) 2 187; –729; 243; –81; ....

   ....

3. (a) Fill in the missing output and input numbers:



What is the term-to-term rule for the output numbers here, + 6 or × 6?

(b) Complete the table below:

Input numbers	1	2	3	4	5		12	x
Output numbers	6			24		36		

NEITHER ADDING NOR MULTIPLYING BY THE SAME NUMBER

1. Consider sequences A to H again and answer the questions that follow:

Sequence A: 2; 5; 8; 11; 14; 17; 20; 23; ...
Sequence B: 4; 5; 8; 13; 20; 29; 40; ...
Sequence C: 1; 2; 4; 8; 16; 32; 64; ...
Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...
Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; ...
Sequence F: 2; 6; 18; 54; 162; 486; ...
Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; ...
Sequence H: 2; 4; 8; 16; 32; 64; ....

(a) Which other sequence(s) is/are of the same kind as sequence B? Explain.

....

(b) In what way are sequences B and E different from the other sequences?

....

....

There are sequences where there is neither a constant difference nor a constant ratio between consecutive terms and yet a pattern still exists, as in the case of sequences B and E.

2. Consider the sequence: 10; 17; 26; 37; 50; ...
   (a) Write down the next five numbers in the sequence. ....
   (b) Eric observed that he can calculate the next term in the sequence as follows: 10 + **7** = 17; 17 + **9** = 26; 26 + **11** = 37. Use Eric's method to check whether your numbers in question (a) above are correct.

   ....

   ....

3. Which of the statements below can Eric use to describe the relationship between the numbers in the sequence in question 2? Test the rule for the first three terms of the sequence and then simply write "yes" or "no" next to each statement.
   (a) Increase the difference between consecutive terms by 2 each time ....

   ....

   ....

   ....

   (b) Increase the difference between consecutive terms by 1 each time ....

   ....

   ....

   ....

   (c) Add two more than you added to get the previous term ....

   ....

   ....

   ....

4. Provide a rule to describe the relationship between the numbers in the sequences below. Use your rule to provide the next five numbers in the sequence.
   (a) 1; 4; 9; 16; 25; ....

   ....

   (b) 2; 13; 26; 41; 58; ....

   ....

   (c) 4; 14; 29; 49; 74; ....

   ....

   (d) 5; 6; 8; 11; 15; 20; ....

   ....

4.2 The position–term relationship in a sequence

USING POSITION TO MAKE PREDICTIONS

1. Take another look at sequences A to H. Which sequence(s) are of the same kind as sequence A? Explain.

Sequence A: 2; 5; 8; 11; 14; 17; 20; 23;...

Sequence B: 4; 5; 8; 13; 20; 29; 40; ...

Sequence C: 1; 2; 4; 8; 16; 32; 64; ...

Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...

Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; ...

Sequence F: 2; 6; 18; 54; 162; 486; ...

Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; ...

Sequence H: 2; 4; 8; 16; 32; 64; ...

....

Sizwe has been thinking about Amanda and Tamara's explanations of how they worked out the rule for sequence A and has drawn up a table. He agrees with them but says that there is another rule that will also work. He explains:

My table shows the terms in the sequence and the difference between consecutive terms:

	1st term	2nd term	3rd term	4th term						
A:	5	8	11	14						
differences	+3	+3	+3	+3	+3	+3	+3	+3	+3	

Sizwe reasons that the following rule will also work:

Multiply the position of the number by 3 and add 2 to the answer.

I can write this rule as a number sentence: ***Position of the number*** *× 3 + 2*

I use my number sentence to check: ***1*** *× 3 + 2 = 5;* ***2*** *× 3 + 2 = 8;* ***3*** *× 3 + 2 = 11*

2. (a) What do the numbers in bold in Sizwe's number sentence stand for?

....

(b) What does the number 3 in Sizwe's number sentence stand for?

....

3. Consider the sequence 5; 8; 11; 14; ...
   Apply Sizwe's rule to the sequence and determine:
   (a) term number 7 of the sequence (b) term number 10 of the sequence

   ....
   (c) the 100th term of the sequence

   ....

4. Consider the sequence: 3; 5; 7; 9; 11; 13; 15; 17; 19;..
   (a) Use Sizwe's explanation to find a rule for this sequence.

   ....
   (b) Determine the 28th term of the sequence.

   ....

MORE PREDICTIONS

Complete the tables below by calculating the missing terms.

1.

Position in sequence	1	2	3	4	10	54
Term	4	7	10	13		

....

....

....

....

2.

Position in sequence	1	2	3	4	8	16
Term	4	9	14	19		

....

....

....

....

3.

Position in sequence	1	2	3	4	7	30
Term	3	15	27			

....

....

....

....

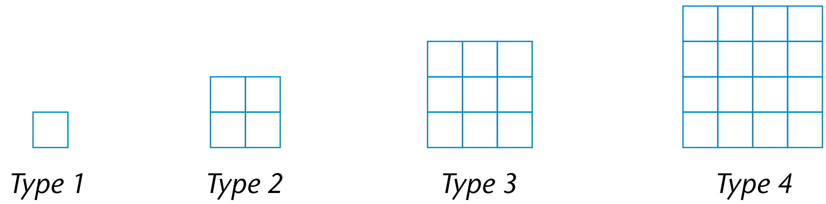
4. Use the rule **Position in the sequence × (position in the sequence + 1)** to complete the table below.

Position in sequence	1	2	3	4	5	6
Term	2					

4.3 Investigating and extending geometric patterns

SQUARE NUMBERS

A factory makes window frames. Type 1 has one windowpane, type 2 has four windowpanes, type 3 has nine windowpanes, and so on.



1. How many windowpanes will there be in type 5? ....
2. How many windowpanes will there be in type 6? ....
3. How many windowpanes will there be in type 7? ....
4. How many windowpanes will there be in type 12? Explain.

....

....

5. Complete the table. Show your calculations.

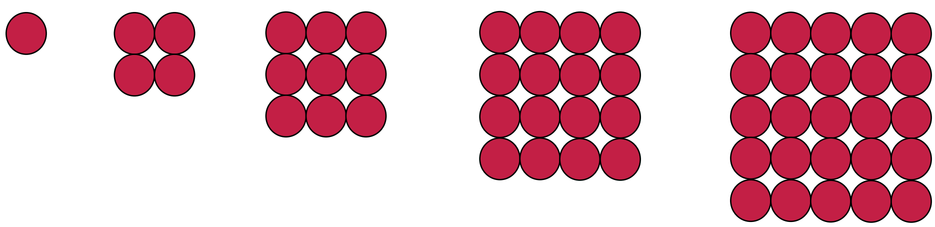
Frame type	1	2	3	4	15	20
Number of windowpanes	1	4	9	16		

....

In algebra we think of a square as a number that is obtained by multiplying a number by itself. So 1 is also a square because 1 × 1 = 1.

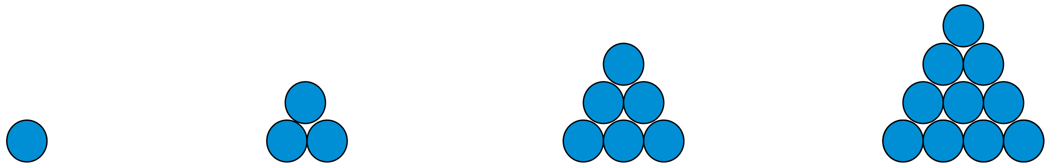
The symbol n is used below to represent the *position number* in the expression that gives the rule (n^2) when generalising.

1^2 2^2 3^2 4^2 5^2 ... n^2



TRIANGULAR NUMBERS

Therese uses circles to form a pattern of triangular shapes:



Picture 1 *Picture 2* *Picture 3* *Picture 4*

1. If the pattern is continued, how many circles must Therese have
   (a) in the bottom row of picture 5? ....
   (b) in the second row from the bottom of picture 5? ....
   (c) in the third row from the bottom of picture 5? ....
   (d) in the second row from the top of picture 5? ....
   (e) in the top row of picture 5? ....
   (f) in total in picture 5? Show your calculation.

....

....

2. How many circles does Therese need to form triangle picture 7?
   Show the calculation.

   ....

3. How many circles does Therese need to form triangle picture 8?

   ....

4. Complete the table below. Show all your work.

Picture number	1	2	3	4	5	6	12	15
Number of circles	1	3	6	10				

   ....

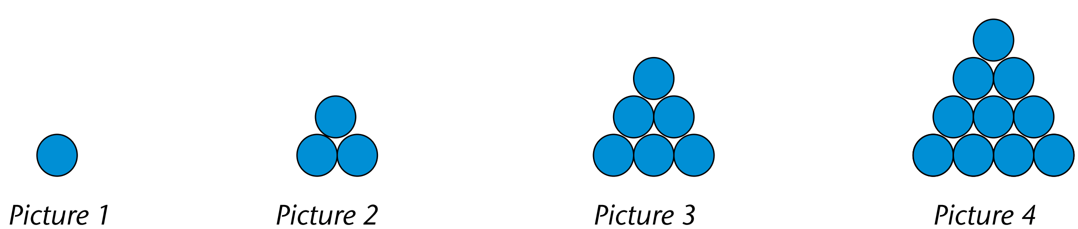
   ....

   ....

   ....

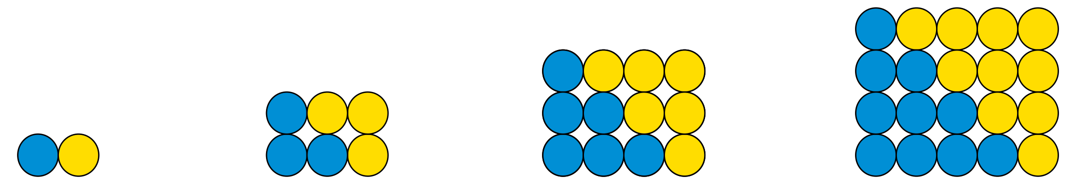
More than 2 500 years ago, Greek mathematicians already knew that the numbers 3, 6, 10, 15 and so on could form a triangular pattern. They represented these numbers with dots which they arranged in such a way that they formed equilateral triangles, hence the name **triangular numbers**. Algebraically we think of them as sums of consecutive natural numbers starting with 1.

Let us revisit the activity on triangular numbers that we did on the previous page.



So far, we have determined the number of circles in the pattern by adding consecutive natural numbers. If we were asked to determine the number of circles in picture 200, for example, it would take us a very long time to do so. We need to find a quicker method of finding any triangular number in the sequence.

Consider the arrangement below.



We have added the yellow circles to the original blue circles and then rearranged the circles in such a way that they are in a rectangular form.

5. Picture 2 is 3 circles long and 2 circles wide. Complete the following sentences:
   (a) Picture 3 is ...... circles long and ...... circles wide.
   (b) Picture 1 is ...... circles long and ...... circle wide.
   (c) Picture 4 is ...... circles long and ...... circles wide.
   (d) Picture 5 is ...... circles long and ...... circles wide.

6. How many circles will there be in a picture that is:
   (a) 10 circles long and 9 circles wide? ....
   (b) 7 circles long and 6 circles wide? ....
   (c) 6 circles long and 5 circles wide? ....
   (d) 20 circles long and 19 circles wide? ....

Suppose we want to have a quicker method to determine the number of circles in picture 15. We know that picture 15 is 16 circles long and 15 circles wide. This gives a total of $15 \times 16 = 240$ circles. But we must compensate for the fact that the yellow circles were originally not there by halving the total number of circles. In other words, the original figure has $240 \div 2 = 120$ circles.

7. Use the above reasoning to calculate the number of circles in:
   (a) picture 20

   ....

   ....

   (b) picture 35

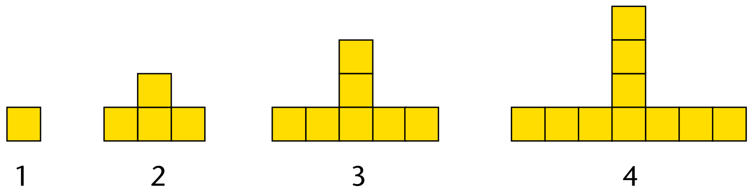
   ....

   ....

4.4 Describing patterns in different ways

T-SHAPED NUMBERS...

The pattern below is made from squares.



1. (a) How many squares will there be in pattern 5? ....

   (b) How many squares will there be in pattern 15? ....

   (c) Complete the table.

Pattern number	1	2	3	4	5	6	20
Number of squares	1	4	7	10			

Below are three different methods or plans to calculate the number of squares for pattern 20. Study each one carefully.

Plan A:

To get from 1 square to 4 squares, you have to add 3 squares. To get from 4 squares to 7 squares, you have to add 3 squares. To get from 7 squares to 10 squares, you have to add 3 squares. So continue to add 3 squares for each pattern until pattern 20.

Plan B:

Multiply the pattern number by 3, and subtract 2. So pattern 20 will have $20 \times 3 - 2$ squares.

Plan C:

The number of squares in pattern 5 is 13. So pattern 20 will have $13 \times 4 = 52$ squares because $20 = 5 \times 4$.

2. (a) Which method or plan (A, B or C) will give the right answer? Explain why.

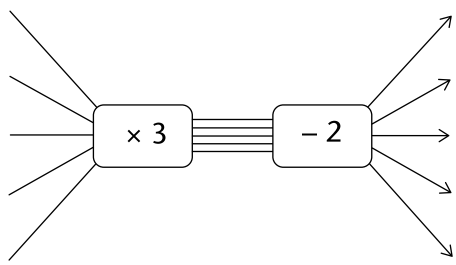
   ....

   (b) Which of the above plans did you use? Explain why?

   ....

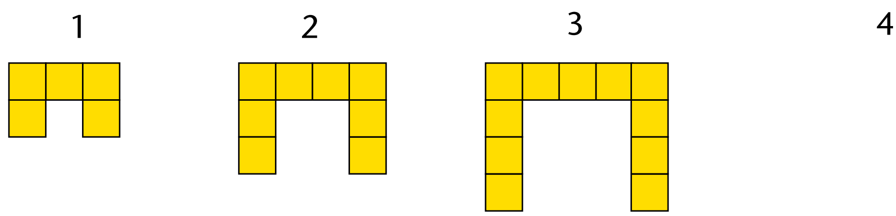
   ....

(c) Can this flow diagram be used to calculate the number of squares? ....



... AND SOME OTHER SHAPES

1. Three figures are given below. Draw the next figure in the tile pattern.



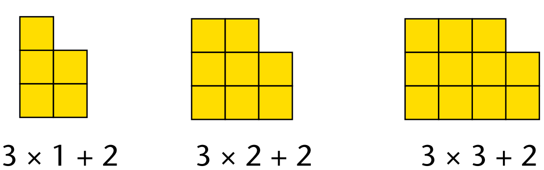
2. (a) If the pattern is continued, how many tiles will there be in the 17th figure? Answer this question by analysing what happens.

....

....

....

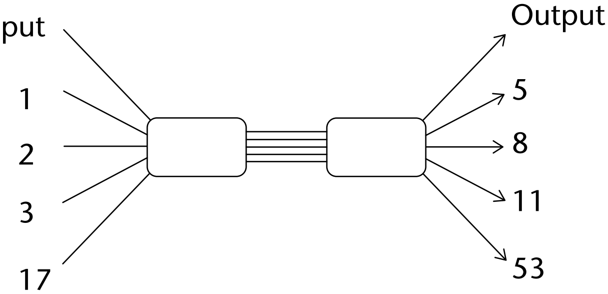
(b) Thato decides that it easier for him to see the pattern when the tiles are rearranged as shown here:



Use Thato's method to determine the number of tiles in the 23rd figure.

....

(c) Complete the flow diagram below by writing the appropriate operators so that it can be used to calculate the number of tiles in any figure of the pattern.



(d) How many tiles will there be in the 50th figure if the pattern is continued?

....

WORKSHEET

1. Write down the next four terms in each sequence. Also explain, in each case, how you figured out what the terms are.

   (a) 2; 4; 8; 14; 22; 32; 44; ....

   ....

   (b) 2; 6; 18; 54; 162; ....

   ....

   (c) 1; 7; 13; 19; 25; ....

   ....

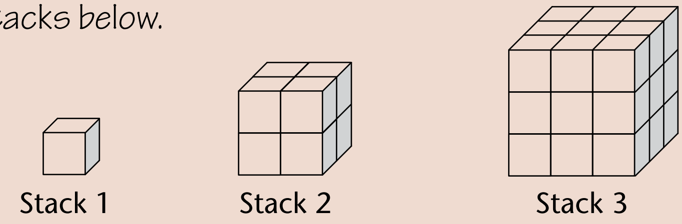
2. (a) Complete the table below by calculating the missing terms.

Position in sequence	1	2	3	4	5	7	10
Term	3	10	17				

   (b) Write the rule to calculate the term from the position in the sequence in words.

   ....

3. Consider the stacks below.



   (a) How many cubes will there be in stack 5? ....

   (b) Complete the table.

Stack number	1	2	3	4	5	6	10
Number of cubes	1	8	27				

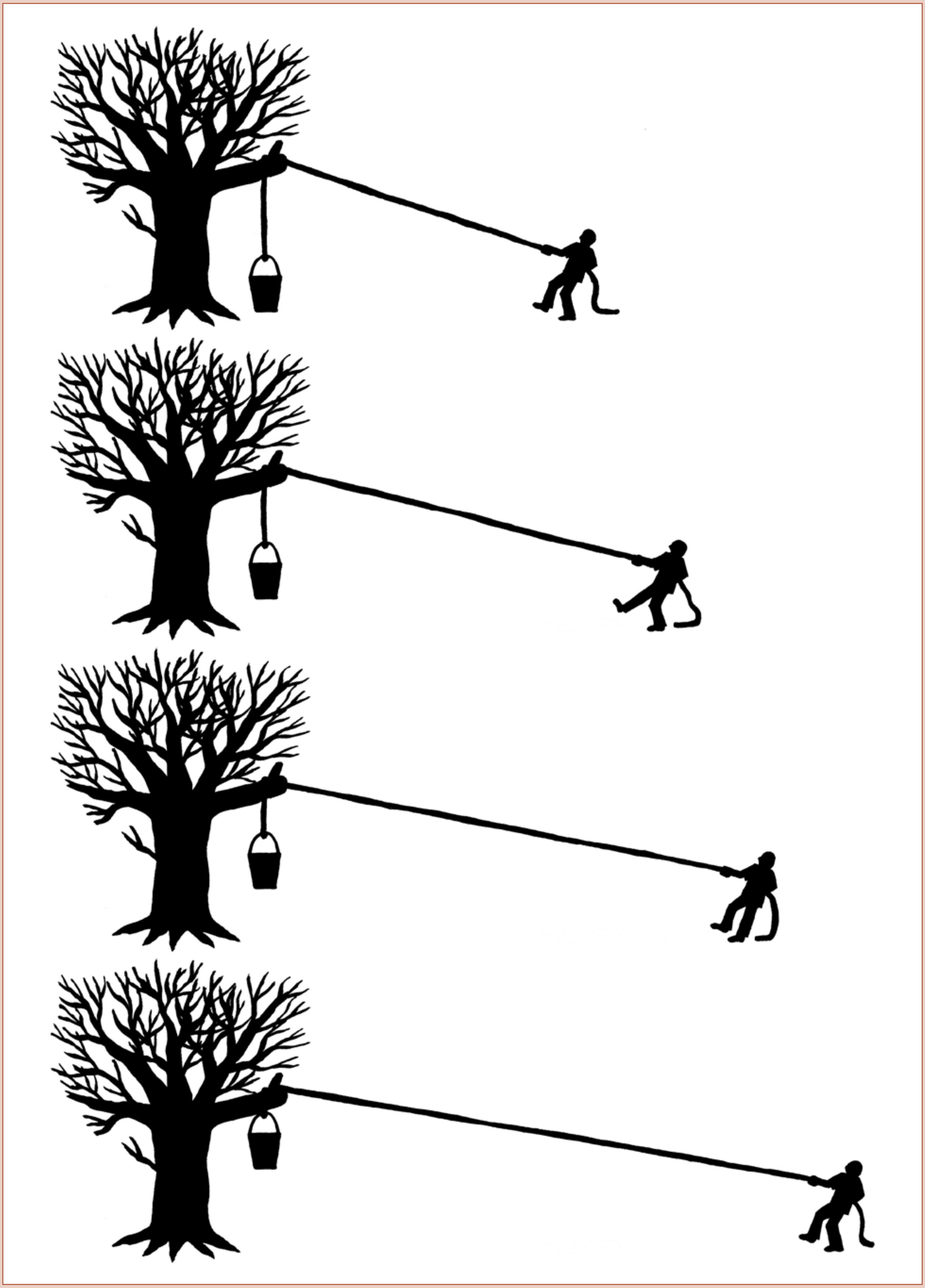
   (c) Write down the rule to calculate the number of cubes for any stack number.

   ....

CHAPTER 5
Functions and relationships

In this chapter you will learn about quantities that change, such as the height of a tree. As the tree grows, the height changes. A quantity that changes is called a variable quantity or just a **variable**.

It is often the case that when one quantity changes, another quantity also changes. For example, as you make more and more calls on a phone, the total cost increases. In this case, we say there is a **relationship** between the amount of money you have to pay and the number of calls you make. You will learn how to describe a relationship between two quantities in different ways.



5 Functions and relationships

5.1 Constant and variable quantities

LOOKING FOR CONNECTIONS BETWEEN QUANTITIES

Consider the following seven situations. There are two quantities in each situation. For each quantity, state whether it is constant (always the same number) or whether it changes. Also state, in each case, whether one quantity has an influence on the other. If it has, try to say how the one quantity will influence the other quantity.

1. Your age and the number of fingers on your hands

   ....

   ....

2. The number of calls you make and the airtime left on your cellphone

   ....

   ....

3. The length of your arm and your ability to finish Mathematics tests quickly

   ....

   ....

4. The number of identical houses to be built and the number of bricks required

   ....

   ....

5. The number of learners at a school and the length of the school day

   ....

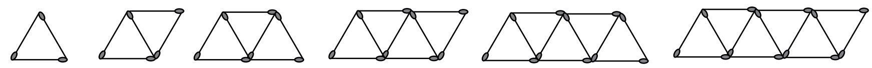
   ....

6. The number of learners at a school and the number of classrooms needed

   ....

   ....

7. The number of matches in each arrangement here, and the number of triangles in the arrangement



If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. It is sometimes possible to find out what value of the one quantity, in other words what number, is linked to a specific value of the other quantity.

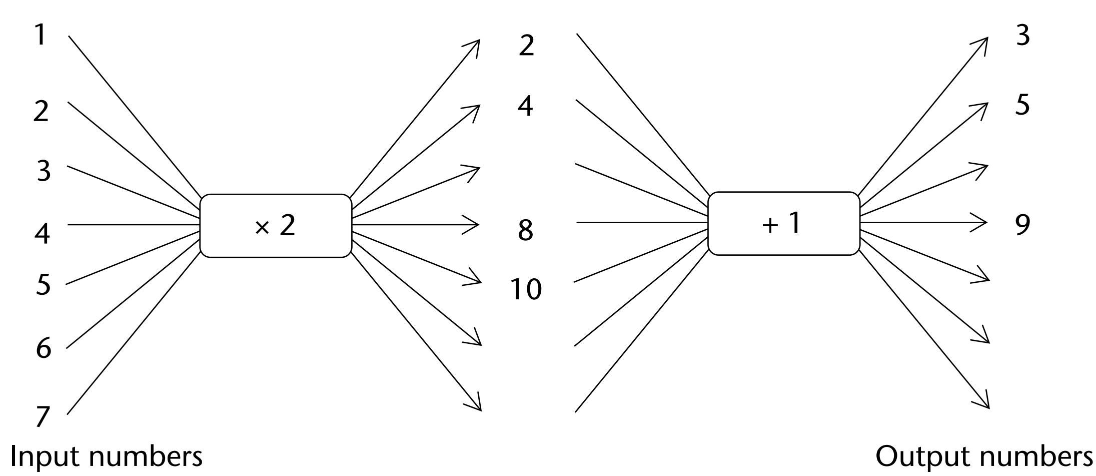
A quantity that changes is called a **variable quantity** or just a **variable**.

8. (a) Look at the match arrangements in question 7. If you know that there are 3 triangles in an arrangement, can you say with certainty how many matches there are in that specific arrangement?

   (b) How many matches are there in the arrangement with 10 triangles?

   (c) Is there another possible answer for question (b)?

9. Complete the flow diagram by filling in all the missing numbers. Do you see any connections between the situation in question 7 and this flow diagram? If so, describe the connections.

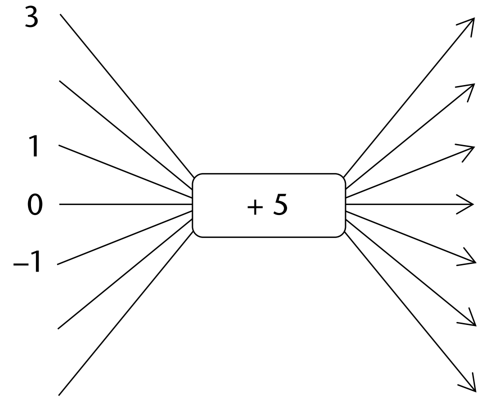


COMPLETING SOME FLOW DIAGRAMS

A relationship between two quantities can be shown with a flow diagram, such as those below. Unfortunately, only some of the numbers can be shown on a flow diagram.

1. Calculate the output numbers for the flow diagram below. Some input numbers are missing. Choose and insert your own input numbers.

(a)



Each input number in a flow diagram has a corresponding **output number**. The first (top) **input number** corresponds to the first output number. The second input number corresponds to the second output number and so on.

We call +5 the **operator**.

(b) What type of numbers are the given input numbers?

....

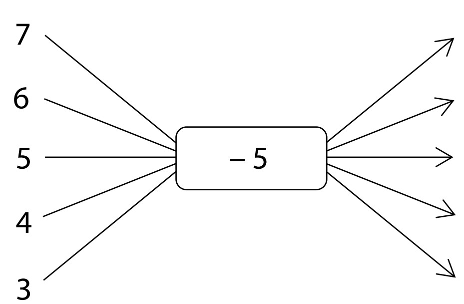
(c) In the above flow diagram, the output number 8 corresponds to the input number 3. Complete the following sentences:

In the relationship shown in the above flow diagram, the output number ..... corresponds to the input number –1.

The input number ..... corresponds to the output number 7.

If more places are added to the flow diagram, the input number ..... will correspond to the output number 31.

2. (a) Complete this flow diagram.

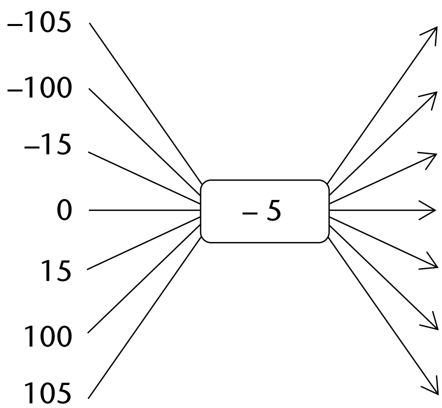


(b) Compare this flow diagram to the flow diagram in question 1. What link do you find between the two?

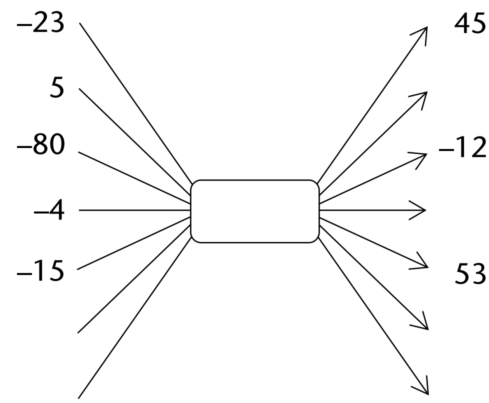
....

3. Complete the flow diagrams below. You have to find out what the **operator** for (b) is, and fill it in yourself.

(a)



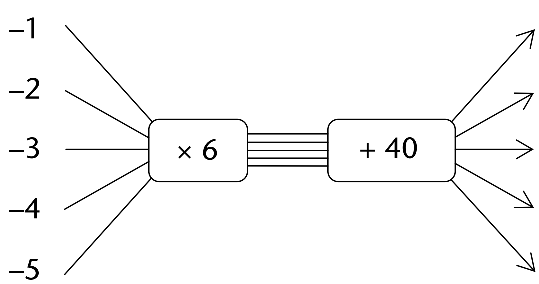
(b)



(c) What number can you add in (a), instead of subtracting 5, that will produce the same output numbers? ....

(d) What number can you subtract in (b), instead of adding a number, that will produce the same output numbers? ....

4. Complete the flow diagram:



A completed flow diagram shows two kinds of information:

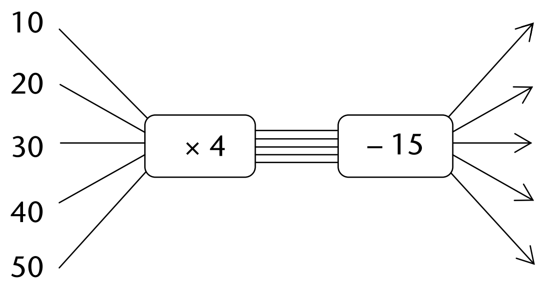
- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

The flow diagram that you have completed in question 4 shows the following information:

- Each input number is multiplied by 6, then 40 is added to produce the output numbers.
- The input and output numbers are connected as shown in the table below.

Input numbers	–1	–2	–3	–4	–5
Output numbers	34	28	22	16	10

5. (a) Describe in words how the output numbers below can be calculated.

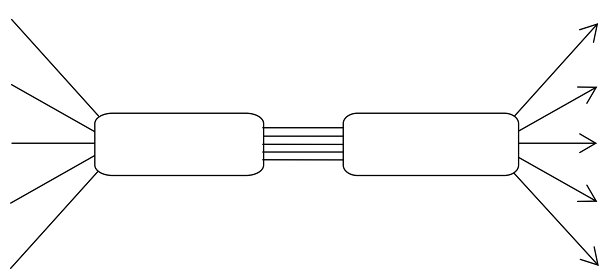


(b) Use the table below to show which output numbers are connected to which input numbers in the above flow diagram.

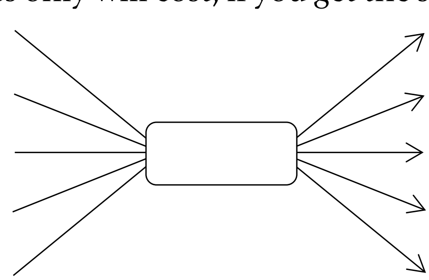
6. The following information is available about the floor space and cost of new houses in a new development. The cost of an empty stand is R180 000.

Floor space in square metres	90	120	150	180	210
Cost of house and stand	540 000	660 000	780 000	900 000	1 020 000

(a) Represent the above information on the flow diagram below.



(b) Show what the houses only will cost, if you get the stand for free.

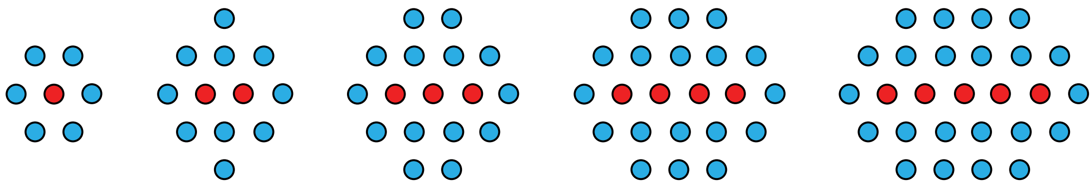


(c) Try to figure out what the cost of a house and stand will be, if there are exactly one hundred 1 m by 1 m sections of floor space in the house.

5.2 Different ways to describe relationships

A RELATIONSHIP BETWEEN RED DOTS AND BLUE DOTS

Here is an example of a relationship between two quantities:



In each arrangement there are some red dots and some blue dots.

1. How many blue dots are there if there is one red dot? ....
2. How many blue dots are there if there are two red dots? ....
3. How many blue dots are there if there are three red dots? ....
4. How many blue dots are there if there are four red dots? ....
5. How many blue dots are there if there are five red dots? ....
6. How many blue dots are there if there are six red dots? ....
7. How many blue dots are there if there are seven red dots? ....
8. How many blue dots are there if there are ten red dots? ....
9. How many blue dots are there if there are twenty red dots? ....
10. How many blue dots are there if there are one hundred red dots? ....

11. Which of the descriptions on the next page correctly describe the relationship between the number of blue dots and the number of red dots in the above arrangements? Test each description thoroughly for all the above arrangements. List them on the dotted line below. Write only the letters, for example (d).

    ....

Something to think about

Are there different possibilities for the number of blue dots if there are 3 red dots in the arrangement?

Are there different possibilities for the number of blue dots if there are 2 red dots in the arrangement?

Are there different possibilities for the number of blue dots if there are 20 red dots in the arrangement?

(a) the number of red dots → $\boxed{\times 4}$ → $\boxed{+ 2}$ → the number of blue dots

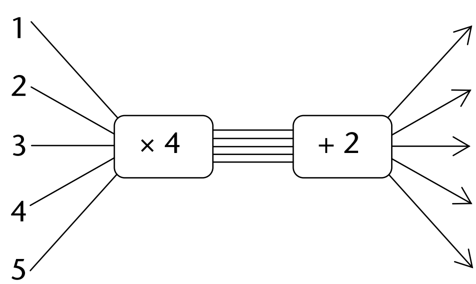
(b) to calculate the number of blue dots you multiply the number of red dots by 2, add 1 and multiply the answer by 2

(c) number of blue dots = 2 × the number of red dots + 4

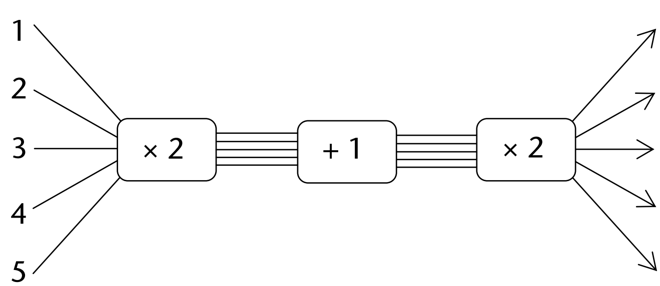
(d)

Number of red dots	1	2	3	4	5	6
Number of blue dots	6	10	14	18	22	26

(e)



(f)



(g) number of blue dots = 4 × the number of red dots + 2

(h) number of blue dots = 2 × (2 × the number of red dots + 1)
(Remember that the calculations inside the brackets are done first.)

The descriptions in (c), (g) and (h) above are called **word formulae**.

TRANSLATING BETWEEN DIFFERENT LANGUAGES OF DESCRIPTION

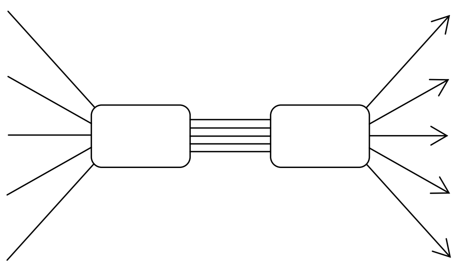
A relationship between two quantities can be described in different ways, including the following:

- a table of values of the two quantities
- a flow diagram
- a word formula
- a symbol formula (or symbolic formula)

You will learn about symbolic formulae in section 5.3.

1. The relationship between two quantities is described as follows:
   The second quantity is always 3 times the first quantity plus 8.
   The first quantity varies between 1 and 5, and it is always a whole number.

   (a) Describe this relationship with the flow diagram.



   (b) Describe the relationship between the two quantities with this table.

   (c) Describe the relationship between the two quantities with a word formula.

....

2. The relationship between two quantities is described as follows:
   The input numbers are the first five odd numbers.

   value of the one quantity → $\boxed{+\ 5}$ → $\boxed{\times\ 3}$ → *the corresponding value of the other quantity*

   (a) Describe this relationship with a table.

   (b) Describe the relationship with a word formula.

....

5.3 Algebraic symbols for variables and relationships

DESCRIBING PROCEDURES IN DIFFERENT WAYS

1. In each case do four things:
   - Complete the table.
   - Describe the relationship with a word formula.
   - Describe the input numbers in words.
   - Describe the output numbers in words.

   (a) input number $\longrightarrow \boxed{\times 10} \longrightarrow \boxed{+ 15} \longrightarrow$ output number

Input number	5	10	15	20	25	30
Output number						

the output number = ...

....

....

   (b) input number $\longrightarrow \boxed{+ 15} \longrightarrow \boxed{\times 10} \longrightarrow$ output number

Input number	5	10	15	20	25	30
Output number						

....

....

....

   (c) input number $\longrightarrow \boxed{\times 2} \longrightarrow \boxed{+ 3} \longrightarrow \boxed{\times 5} \longrightarrow$ output number

Input number	5	10	15	20	25	30
Output number						

....

....

....

Formulae with symbols

Instead of writing "input number" and "output number" in formulae, one may just write a single letter symbol as an abbreviation.

Mathematicians have long ago adopted the convention of using the letter symbol x as an abbreviation for "input number", and the letter symbol y as an abbreviation for "output number".

Other letter symbols than x and y are also used to indicate variable quantities.

The word formula you wrote for question 1(a) can be written more shortly as

$$y = 10 \times x + 15$$

Mathematicians have also long ago agreed that one may leave the ×-sign out when writing **symbolic formulae**.

It is not at all wrong to use the multiplication sign in symbolic formulae.

So, instead of $y = 10 \times x + 15$ we may write $y = 10x + 15$.

2. Rewrite your word formulae in questions 1(b) and 1(c) as symbolic formulae.

   ....

3. Write a word formula for each of the following relationships:

   (a) $y = 7x + 10$ ....

   (b) $y = 7(x + 10)$ ....

   (c) $y = 7(2x + 10)$ ....

WRITING SYMBOLIC FORMULAE

Describe each of the following relationships with a symbolic formula:

1. To calculate the output number, the input number is multiplied by 4 and 7 is subtracted from the answer.

   ....

2. To calculate the output number, 7 is subtracted from the input number and the answer is multiplied by 5.

   ....

3. To calculate the output number, 7 is subtracted from the input number, the answer is multiplied by 5 and 3 is added to this answer.

   ....

Chapter 6
Algebraic expressions 1

An algebraic expression is a description of certain calculations that have to be done in a certain order. In this chapter, you will be introduced to the language of algebra. You will also learn about expressions that appear to be different but that produce the same results when evaluated. When we evaluate an expression, we choose or are given a value of the variable in the expression. Because now we have an actual value, we can carry out the operations (+, −, ×, ÷) in the expression using this value.

x y z

yxz

z y x

y x z

6 Algebraic expressions 1

6.1 Algebraic language

WORDS, DIAGRAMS AND SYMBOLS

1. Complete this table.

	Words	Flow diagram	Expression
	Multiply a number by two and add six to the answer.	$\rightarrow \boxed{\times 2} \rightarrow \boxed{+ 6} \rightarrow$	$2 \times x + 6$
(a)	Add three to a number and then multiply the answer by two.		
(b)		$\rightarrow \boxed{\times 5} \rightarrow \boxed{- 1} \rightarrow$	
(c)			$7 + 4 \times x$
(d)			$10 - 5 \times x$

An **algebraic expression** indicates **a sequence of calculations** that can also be described in words or with a flow diagram.

The flow diagram illustrates the **order** in which the calculations must be done.

In algebraic language the **multiplication sign is usually omitted**. So we write $2x$ instead of $2 \times x$. We also write $x \times 2$ as $2x$.

2. Write the following expressions in 'normal' algebraic language:

(a) $-2 \times a + b$ (b) $a2$

....

LOOKING DIFFERENT BUT YET THE SAME

1. Complete the table by calculating the numerical values of the expressions for the values of x. Some answers for $x = 1$ have been done for you as an example.

	x	1	3	7	10
(a)	$2x + 3x$	$2 \times 1 + 3 \times 1$ $2 + 3 = 5$			
(b)	$5x$				
(c)	$2x + 3$				
(d)	$5x^2$	$5 \times (1)^2$ $5 \times 1 = 5$			

2. Do the expressions $2x + 3x$ and $5x$, in question 1 above, produce different answers or the same answer for:
   (a) $x = 3$? (b) $x = 10$?

   ....

3. Do the expressions $2x + 3$ and $5x$ produce different answers or the same answer for:
   (a) $x = 3$? (b) $x = 10$?

   ....

4. Write down all the algebraic expressions in question 1 that have the same numerical value for the same value(s) of x, although they may look different. Justify your answer.

   ....

   ....

   ....

   ....

   ....

One of the things we do in algebra is to **evaluate** expressions. When we evaluate an expression we choose or are given a value of the variable in the expression. Because now we have an actual value we can carry out the operations in the expression using this value, as in the examples given in the table.

Algebraic expressions that have the same numerical value for the same value of x but look different are called **equivalent expressions**.

5. Say whether the following statements are true or false. Explain your answer in each case.
   (a) The expressions $2x + 3x$ and $5x$ are equivalent.

   ....

   ....

   (b) The expressions $2x + 3$ and $5x$ are equivalent.

   ....

   ....

6. Consider the expressions $3x + 2z + y$ and $6xyz$.

   Remember that $6xyz$ is the same as $6 \times x \times y \times z$.

   (a) What is the value of $3x + 2z + y$ for $x = 4$, $y = 7$ and $z = 10$?

   ....

   ....

   (b) What is the value of $6xyz$ for $x = 4$, $y = 7$ and $z = 10$?

   ....

   ....

   (c) Are the expressions $3x + 2z + y$ and $6xyz$ equivalent? Explain.

   ....

   ....

To show that the two expressions in question 5(a) are equivalent we write $2x + 3x = 5x$.

The term $3x$ is a product. The number 3 is called the **coefficient** of x.

We can explain why this is so:

$2x + 3x = (x + x) + (x + x + x) = 5x$

We say the expression $2x + 3x$ **simplifies** to $5x$.

7. In each case below, write down an expression equivalent to the one given.

(a) $3x + 3x$ ....

(b) $3x + 8x + 2x$ ....

(c) $8b + 2b + 2b$ ....

(d) $7m + 2m + 10m$ ....

(e) $3x^2 + 3x^2$ ....

(f) $3x^2 + 8x^2 + 2x^2$ ....

8. What is the coefficient of x^2 for the expression equivalent to $3x^2 + 8x^2 + 2x^2$?

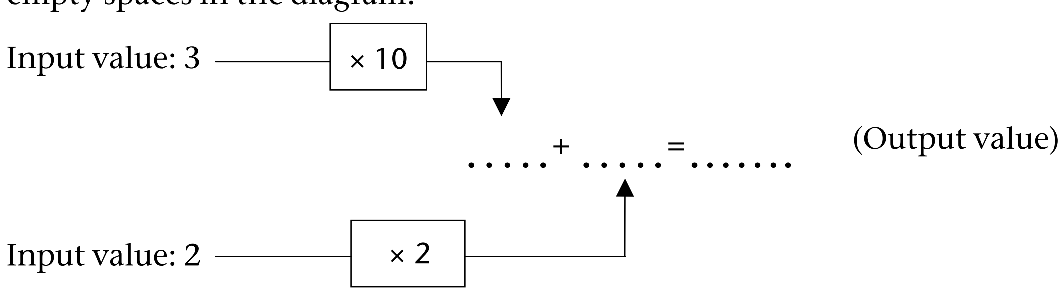
....

In an expression that can be written as a sum, the different parts of the expression are called the **terms of the expression**. For example, $3x$, $2z$ and y are the terms of the expression $3x + 2z + y$.

An expression can have **like terms** or **unlike terms** or both.

Like terms are terms that have the **same variable(s) raised to the same power**. The terms $2x$ and $3x$ are examples of like terms.

9. (a) Calculate the numerical value of $10x + 2y$ for $x = 3$ and $y = 2$ by completing the empty spaces in the diagram.



(b) What is the output value for the expression $12xy$ for $x = 3$ and $y = 2$?

....

(c) Are the expressions $10x + 2y$ and $12xy$ equivalent? Explain.

....

....

(d) Are the terms $10x$ and $2y$ like or unlike terms? Explain.

....

10. (a) Which of the following algebraic expressions do you think will give the same results?

A. $6x + 4x$ B. $10x$ C. $10x^2$ D. $9x + x$

....

(b) Test the algebraic expressions you have identified for the following values of x:

$x = 10$ $x = 17$ $x = 54$

....	....	....
....	....	....
....	....	....
....	....	....
....	....	....
....	....	....

(c) Are the terms $6x$ and $4x$ like or unlike terms? Explain.

....

....

(d) Are the terms $10x$ and $10x^2$ like or unlike terms? Explain.

....

....

....

11. Ashraf and Hendrik have a disagreement about whether the terms $7x^2y^3$ and $301y^3x^2$ are like terms or not. Hendrik thinks they are not, because in the first term the x^2 comes before the y^3 whereas in the second term the y^3 comes before the x^2. Explain to Hendrik why his argument is not correct.

....

....

....

....

....

12. Explain why the terms $5abc$, $10acb$ and $15cba$ are like terms.

....

6.2 Add and subtract like terms

REARRANGE TERMS AND THEN COMBINE LIKE TERMS

1. Complete the table by evaluating the expressions for the given values of x.

x	1	2	10
$30x + 80$	$30 \times 1 + 80$ $= 30 + 80 = 110$		
$5x + 20$			
$30x + 80 + 5x + 20$			
$35x + 100$			
$135x$			

2. Write down all the expressions in the table that are equivalent.

....

3. Tim thinks that the expressions $135x$ and $35x + 100$ are equivalent because for $x = 1$ they both have the same numerical value 135.
   Explain to Tim why the two expressions are not equivalent.

....

....

....

We have already come across the commutative and associative properties of operations. We will now use these properties to help us form equivalent algebraic expressions.

Commutative property
The order in which we add or multiply numbers does not change the answer: $a + b = b + a$ and $ab = ba$

Associative property

The way in which we group three or more numbers when adding or multiplying does not change the answer: $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$

We can find an equivalent expression by **rearranging** and **combining like terms**, as shown below:

$30x + 80 + 5x + 20$

Hence $30x + (80 + 5x) + 20$

Hence $30x + (5x + 80) + 20$

$= (30x + 5x) + (80 + 20)$

$= 35x + 100$

The terms 80 and 20 are called **constants**. The numbers 30 and 5 are called **coefficients**.

Brackets are used in the expression on the left to show how the like terms have been rearranged.

Like terms are combined to form a single term.

The terms $30x$ and $5x$ are combined to get the new term $35x$, and the terms 80 and 20 are combined to form the new term 100. We say that **the expression** $30x + 80 + 5x + 20$ **is simplified** to a new expression $35x + 100$.

4. Simplify the following expressions:

(a) $13x + 7 + 6x - 2$

....

....

(b) $21x - 8 + 7x + 15$

....

....

(c) $18c - 12d + 5c - 7c$

....

....

(d) $3abc + 4 + 7abc - 6$

....

....

(e) $12x^2 + 2x - 2x^2 + 8x$

....

....

(f) $7m^3 + 7m^2 + 9m^3 + 1$

....

....

When you are not sure about whether you correctly simplified an expression, it is always advisable to check your work by evaluating the original expression and the simplified expression for some values. This is a very useful habit to have.

When we use a value of the variable in the expression we call that **substitution**.

5. Make a simpler expression that is equivalent to the given expression. Test your answer for three different values of x, and redo your work until you get it right.
   (a) Simplify $(15x + 7y) + (25x + 3 + 2y)$ (b) Simplify $12mn + 8mn$

....

....

....

In questions 6 to 8 below, write down the letter representing the correct answer. Also explain why you think your answer is correct.

6. The sum of $5x^2 + x + 7$ and $x - 9$ is:
   A. $x^2 - 2$ B. $5x^2 + 2x + 16$ C. $5x^2 + 16$ D. $5x^2 + 2x - 2$

....

7. The sum of $6x^2 - x + 4$ and $x^2 - 5$ is equivalent to:
   A. $7x^2 - x + 9$ B. $7x^2 - x - 1$ C. $6x^4 - x - 9$ D. $7x^4 - x - 1$

....

8. The sum of $5x^2 + 2x + 4$ and $3x^2 - 5x - 1$ can be expressed as:
   A. $8x^2 + 3x + 3$ B. $8x^2 + 3x - 3$ C. $8x^2 - 3x + 3$ D. $8x^2 - 3x - 3$

....

Combining like terms is a useful algebraic habit. It allows us to replace an expression with another expression that may be convenient to work with.

Do the following questions to get a sense of what we are talking about.

CONVENIENT REPLACEMENTS

1. Consider the expression $x + x + x + x + x + x + x + x + x + x$. What is the value of the expression in each of the following cases?
   (a) $x = 2$ (b) $x = 50$

....

....

2. Consider the expression $x + x + x + z + z + y$. What is the value of the expression in each of the following cases?
   (a) $x = 4, y = 7, z = 10$ (b) $x = 0, y = 8, z = 22$

....

....

....

3. Suppose you have to evaluate $3x + 7x$ for $x = 20$. Will calculating 10×20 give the correct answer? Explain.

....

....

....

....

....

Suppose we evaluate the expression $3x + 7x$ for $x = 20$ without first combining the like terms. We will have to do **three** calculations, namely 3×20, then 7×20 and then find the sum of the two: $3 \times 20 + 7 \times 20 = 60 + 140 = 200$.

But if we first combine the like terms $3x$ and $7x$ into one term $10x$, we only have to do **one calculation**: $10 \times 20 = 200$. This is one way of thinking about the convenience or usefulness of simplifying an algebraic expression.

4. The expression $5x + 3x$ is given and you are required to evaluate it for $x = 8$. Will calculating 8×8 give the correct answer? Explain.

....

....

....

....

....

5. Suppose you have to evaluate $7x + 5$ for $x = 10$. Will calculating 12×10 give the correct answer? Explain.

....

....

....

6. The expression $5x + 3$ is given and you have to evaluate it for $x = 8$. Will calculating 8×8 give the correct answer? Explain.

....

Samantha was asked to evaluate the expression $12x^2 + 2x - 2x^2 + 8x$ for $x = 12$. She thought to herself that just substituting the value of x directly into the terms would require a lot of work. She first combined the like terms as shown below:

$12x^2 \mathbf{- 2x^2 + 2x} + 8x$

$= 10x^2 + 10x$

Then for $x = 10$, Samantha found the value of $10x^2 + 10x$ by calculating

$10 \times 10^2 + 10 \times 10$

$= 1\,000 + 100$

$= 1\,100$

The terms $+2x$ and $-2x^2$ change positions by the commutative property of operations.

Use Samatha's way of thinking for questions 7 to 9.

7. What is the value of $12x + 25x + 75x + 8x$ when $x = 6$?

....

....

8. Evaluate $3x^2 + 7 + 2x^2 + 3$ for $x = 5$.

....

....

9. When Zama was asked to evaluate the expression $2n - 1 + 6n$ for $n = 4$, she wrote down the following:

   $2n - 1 + 6n = n + 6n = 6n^2$

   Hence for $n = 4$: $6 \times (4)^2 = 6 \times 8 = 48$

   Explain where Zama went wrong and why.

....

....

....

....

....

WORKSHEET

1. Complete the table.

	Words	Flow diagram	Expression
(a)	Multiply a number by three and add two to the answer.	$\rightarrow \boxed{\times 3} \rightarrow \boxed{+2} \rightarrow$	
(b)			$9x - 6$
(c)		$\rightarrow \boxed{\times 7} \rightarrow \boxed{-3} \rightarrow$	$7x - 3$

2. Which of the following pairs consist of like terms? Explain.

A. $3y; -7y$ B. $14e^2; 5e$ C. $3y^2z; 17y^2z$ D. $-bcd; 5bd$

....

....

3. Write the following in the 'normal' algebraic way:

(a) $c2 + d3$ (b) $7 \times d \times e \times f$

....

4. Consider the expression $12x^2 - 5x + 3$.

(a) What is the number 12 called? ....

(b) Write down the coefficient of x. ....

(c) What name is given to the number 3? ....

5. Explain why the terms $5pqr$ and $-10prq$ and $15qrp$ are like terms.

....

6. If $y = 7$, what is the value of each of the following?

(a) $y + 8$ (b) $9y$ (c) $7 - y$

....

7. Simplify the following expressions:
   (a) $18c + 12d + 5c - 7c$
   (b) $3def + 4 + 7def - 6$

8. Evaluate the following expressions for $y = 3, z = -1$:
   (a) $2y^2 + 3z$
   (b) $(2y)^2 + 3z$

9. Write each algebraic expression in the simplest form.
   (a) $5y + 15y$
   (b) $5c + 6c - 3c + 2c$
   (c) $4b + 3 + 16b - 5$
   (d) $7m + 3n + 2 - 6m$
   (e) $5h^2 + 17 - 2h^2 + 3$
   (f) $7e^2f + 3ef + 2 + 4ef$

10. Evaluate the following expressions:
   (a) $3y + 3y + 3y + 3y + 3y + 3y$ for $y = 18$
   (b) $13y + 14 - 3y + 6$ for $y = 200$
   (c) $20 - y^2 + 101y^2 + 80$ for $y = 1$
   (d) $12y^2 + 3yz + 18y^2 + 2yz$ for $y = 3$ and $z = 2$

Chapter 7
Algebraic equations 1

In this chapter you will learn to find numbers that make certain statements true. A statement about an unknown number is called an equation. When we work to find out which number will make the equation true, we say we solve the equation. The number that makes the equation true is called the solution of the equation.

4	71	53	8	98	78	54
46	9	6	2	60	81	62
70	6	8	33	2	40	64
27	70	31	63	59	71	62
42	85	32	85	81	51	73
70	64	33	96	32	23	69
82	9	59	54	96	43	29
63	71	86	81	6	29	56
74	21	17	94	6	33	56
18	63	73	76	91	32	39
3	87	23	94	84	75	69
36	49	90	73	62	70	22
10	91	40	92	68	87	57
62	76	72	79	68	25	8
9	72	31	37	37	46	49
48	58	64	92	34	83	95
18	50	88	51	92	89	10
49	49	100	60	60	75	40

7 Algebraic equations 1

7.1 Setting up equations

An **equation** is a mathematical sentence that is true for some numbers but false for other numbers. The following are examples of equations:

$$x + 3 = 11 \quad \text{and} \quad 2^x = 8$$

$x + 3 = 11$ is true if $x = 8$, but false if $x = 3$.

When we look for a number or numbers that make an equation true we say that we are **solving the equation**. For example, $x = 4$ is the **solution** of $2x = 8$ because it makes $2x = 8$ true. (Check: $2 \times 4 = 8$)

LOOKING FOR NUMBERS TO MAKE STATEMENTS TRUE

1. Are the following statements true or false? Justify your answer.
   (a) $x - 3 = 0$, if $x = -3$

   ....

   (b) $x^3 = 8$, if $x = -2$

   ....

   (c) $3x = -6$, if $x = -3$

   ....

   (d) $3x = 1$, if $x = 1$

   ....

   (e) $6x + 5 = 47$, if $x = 7$

   ....

2. Find the original number. Show your reasoning.
   (a) A number multiplied by 10 is 80.

   ....

   (b) Add 83 to a number and the answer is 100.

   ....

   (c) Divide a number by 5 and the answer is 4.

   ....

(d) Multiply a number by 4 and the answer is 20.

....

(e) Twice a number is 100.

....

(f) A number raised to the power 5 is 32.

....

(g) A number raised to the power 4 is –81.

....

(h) Fifteen times a number is 90.

....

(i) 93 added to a number is –3.

....

(j) Half a number is 15.

....

3. Write the equations below in words using "a number" in place of the letter symbol x. Then write what you think "the number" is in each case.
Example: $4 + x = 23$. *Four plus a number equals twenty-three. The number is 19.*

(a) $8x = 72$

....

(b) $\frac{2x}{5} = 2$

....

(c) $2x + 5 = 21$

....

(d) $12 + 9x = 30$

....

(e) $30 - 2x = 40$

....

(f) $5x + 4 = 3x + 10$

....

....

7.2 Solving equations by inspection

THE ANSWER IS IN PLAIN SIGHT

1. Seven equations are given below the table. Use the table to find out for which of the given values of x it will be true that the left-hand side of the equation is equal to the right-hand side.

You can read the solutions of an equation from a table.

x	-3	-2	-1	0	1	2	3	4
$2x + 3$	-3	-1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	-11	-8	-5	-2	1	4	7	10
$10x - 7$	-37	-27	-17	-7	3	13	23	33
$5x + 3$	-12	-7	-2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	-2

(a) $2x + 3 = 5x + 3$

....

(b) $5x + 3 = 9 - x$

....

(c) $2x + 3 = x + 4$

....

(d) $10x - 7 = 5x + 3$

....

(e) $3x - 2 = x + 4$

....

(f) $9 - x = 2x + 3$

....

(g) $10 - 3x = 3x - 2$

....

Two or more equations can have the same solution. For example, $5x = 10$ and $x + 2 = 4$ have the same solution; $x = 2$ is the solution for both equations.

Two equations are called **equivalent** if they have the same solution.

2. Which of the equations in question 1 have the same solutions? Explain.

....

....

....

3. Complete the table below.
   Then answer the questions that follow.

You can also do a search by narrowing down the possible solution to an equation.

x	0	5	10	15	20	25	30	35	40
$2x + 3$									
$3x - 10$									

(a) Can you find a solution for $2x + 3 = 3x - 10$ in the table? ....

(b) What happens to the values of $2x + 3$ and $3x - 10$ as x increases? Do they become bigger or smaller?

....

(c) Is there a point where the value of $3x - 10$ becomes bigger or smaller than the value of $2x + 3$ as the value of x increases? If so, between which x-values does this happen?

This point where the two expressions are equal is called the **break-even point.**

....

(d) Now that you narrowed down where the possible solution can be, try other possible values for x until you find out for what value of x the statement $2x + 3 = 3x - 10$ is true.

"Searching" for the solution of an equation by using tables or by narrowing down to the possible solution is called **solution by inspection**.

....

....

....

7.3 More examples

LOOKING FOR AND CHECKING SOLUTIONS

1. What is the solution for the equations below?

(a) $x - 3 = 4$

....

(b) $x + 2 = 9$

....

(c) $3x = 21$

....

(d) $3x + 1 = 22$

....

When a certain number is the solution of an equation we say that the number **satisfies** the equation. For example, $x = 4$ satisfies the equation $3x = 12$ because $3 \times 4 = 12$.

2. Choose the number in brackets that satisfies the equation. Explain your choice.

(a) $12x = 84$ $\{5; 7; 10; 12\}$

....

(b) $\frac{84}{x} = 12$ $\{-7; 0; 7; 10\}$

....

(c) $48 = 8k + 8$ $\{-5; 0; 5; 10\}$

....

(d) $19 - 8m = 3$ $\{-2; -1; 0; 1; 2\}$

....

(e) $20 = 6y - 4$ $\{3; 4; 5; 6\}$

....

(f) $x^3 = -64$ $\{-8; -4; 4; 8\}$

....

(g) $5^x = 125$ $\{-3; -1; 1; 3\}$

....

(h) $2^x = 8$ $\{1; 2; 3; 4\}$

....

(i) $x^2 = 9$ $\{1; 2; 3; 4\}$

....

3. What makes the following equations true? Check your answers.

(a) $m + 8 = 100$

....

....

(b) $80 = x + 60$

....

....

(c) $26 - k = 0$

....

....

(d) $105 \times y = 0$

....

....

(e) $k \times 10 = 10$

(f) $5x = 100$

....

....

(g) $\frac{15}{t} = 5$

(h) $3 = \frac{t}{5}$

....

....

4. Solve the equations below by inspection. Check your answers.

(a) $12x + 14 = 50$

(b) $100 = 15m + 25$

....

....

....

(c) $\frac{100}{x} = 20$

(d) $7m + 5 = 40$

....

....

....

(e) $2x + 8 = 10$

(f) $3x + 10 = 31$

....

....

(g) $-1 + 2x = -11$

(h) $2 + \frac{x}{7} = 5$

....

....

(i) $100 = 64 + 9x$

(j) $\frac{2x}{6} = 4$

....

....

Term 1
Revision and assessment

Revision

Show all your steps of working.

WHOLE NUMBERS

1. (a) Write both 300 and 160 as products of prime factors.

   ....

   ....

   (b) Determine the HCF and LCM of 300 and 160.

   ....

   ....

2. Tommy, Thami and Timmy are given birthday money by their grandmother in the ratio of their ages. They are turning 11, 13 and 16 years old, respectively. If the total amount of money given to all three boys is R1 000, how much money does Thami get on his birthday?

   ....

   ....

3. Tshepo and his family are driving to the coast on holiday. The distance is 1 200 km and they must reach their destination in 12 hours. After 5 hours, they have travelled 575 km. Then one of their tyres bursts. It takes 45 minutes to get the spare wheel on, before they can drive again. At what average speed must they drive the remainder of the journey to reach their destination on time?

   ....

   ....

   ....

4. The number of teachers at a school has increased in the ratio 5 : 6. If there used to be 25 teachers at the school, how many teachers are there now?

   ....

   ....

5. ABC for Life needs to have their annual statements audited. They are quoted R8 500 + 14% VAT by Audits Inc. How much will ABC for Life have to pay Audits Inc. in total?

....

....

....

....

6. Reshmi invests R35 000 for three years at an interest rate of 8,2% per annum. Determine how much money will be in her account at the end of the investment period.

....

....

....

....

....

7. Lesebo wants to buy a lounge suite that costs R7 999 cash. He does not have enough money and so decides to buy it on hire purchase. The store requires a 15% deposit up front, and 18 monthly instalments of R445.

   (a) Calculate the deposit that Lesebo must pay.

....

....

....

....

   (b) How much extra does Lesebo pay because he buys the lounge suite on hire purchase, rather than in cash?

....

....

....

....

8. Consider the following exchange rates table:

South African Rand	1.00 ZAR	inv. 1.00 ZAR
Euro	*0.075370*	*13.267807*
US Dollar	*0.098243*	*10.178807*
British Pound	*0.064602*	*15.479409*
Indian Rupee	*5.558584*	*0.179902*
Australian Dollar	*0.102281*	*9.776984*
Canadian Dollar	*0.101583*	*9.844200*
Emirati Dirham	*0.360838*	*2.771327*
Swiss Franc	*0.093651*	*10.677960*
Chinese Yuan Renminbi	*0.603065*	*1.658195*
Malaysian Ringgit	*0.303523*	*3.294646*

(a) Write down the amount in rand that needs to be exchanged to get 1 Swiss franc. Give your answer to the nearest cent.

....

(b) Write down the only currency for which an exchange of R100 will give you more than 100 units of that currency.

....

....

(c) Ntsako is travelling to Dubai and converts R10 000 into Emirati dirhams. How many dirhams does Ntsako receive (assume no commission)?

....

INTEGERS

Don't use a calculator for any of the questions in this section.

1. Write a number in each box to make the calculations correct.

(a) $\square + \square = -11$ (b) $\square - \square = -11$

2. Fill <, > or = into each block to show the relationships.

(a) $-23 \ \square \ 20$ (b) $-345 \ \square \ -350$

(c) $4 - 3 \ \square \ 3 - 4$ (d) $5 - 7 \ \square \ -(7 - 5)$

(e) -9×2 $\square$ -9×-2

(f) -4×5 $\square$ 4×-5

(g) $-10 \div 5$ $\square$ $-10 \div -2$

(h) -15×-15 $\square$ 224

3. Follow the pattern to complete the number sequences.

   (a) 8; 5; 2; $\square$

   (b) 2; −4; 8; $\square$

   (c) −289; −293; −297; $\square$

4. Look at the number lines. In each case, the missing number is halfway between the other two numbers. Fill in the correct values in the boxes.

   (a) −456 $\square$ −448

   (b) −11 $\square$ 5

5. Calculate the following:

   (a) $-5 - 7$

   ....

   (b) $7 - 10$

   ....

   (c) $8 - (-9)$

   ....

   (d) $(-5)(-2)(-4)$

   ....

   (e) $5 + 4 \times -2$

   ....

   (f) $\dfrac{(\sqrt{4})(-2)^2}{-4}$

   ....

   (g) $\dfrac{-(-3)^3\sqrt[3]{125}}{(-9)(3)}$

   ....

   ....

   (h) $\dfrac{\sqrt[3]{-64}}{-3-1}$

   ....

   ....

6. (a) Write down two numbers that multiply to give −15. (One of the numbers must be positive and the other negative.)

   ....

   (b) Write down two numbers that add to 15. One of the numbers must be positive and the other negative.

   ....

7. At 5 a.m., the temperature in Kimberley was –3 °C. At 1 p.m., it was 17 °C. By how many degrees had the temperature risen?

....

....

8. A submarine is 220 m below the surface of the sea. It travels 75 m upwards. How far below the surface is it now?

....

....

EXPONENTS

You should not use a calculator for any of the questions in this section.

1. Write down the value of the following:

   (a) $(-3)^3$ ....

   (b) -5^2 ....

   (c) $(-1)^{200}$ ....

   (d) $(10^2)^2$ ....

2. Write the following numbers in scientific notation:

   (a) 200 000 (b) 12,345

   ....

3. Write the following numbers in ordinary notation:

   (a) $1{,}3 \times 10^2$ (b) $7{,}01 \times 10^7$

   ....

4. Which of the following numbers is bigger: $5{,}23 \times 10^{10}$ or $2{,}9 \times 10^{11}$?

....

....

....

5. Simplify the following:

(a) $2^7 \times 2^3$ ……………………………………………………………

(b) $2x^3 \times 4x^4$ ……………………………………………………………

(c) $(-8y^6) \div (4y^3)$ ……………………………………………………………

(d) $(3x^8)^3$ ……………………………………………………………

(e) $(2x^5)(0{,}5x^{-5})$ ……………………………………………………………

(f) $(-3a^2b^3c)(-4abc^2)^2$ ……………………………………………………………

(g) $\dfrac{(2xy^2z^3)(-5y^2z)^2}{20xy^8z^4}$ ……………………………………………………………

6. Write down the values of each of the following:

(a) $(0{,}6)^2$ …………………… (b) $(0{,}2)^3$ ……………………

(c) $\left(\dfrac{1}{2}\right)^5$ …………………… (d) $\sqrt{\dfrac{1}{4}}$ ……………………

(e) $4\sqrt{\dfrac{9}{64}}$ …………………… (f) $\sqrt[3]{0{,}001}$ ……………………

NUMERIC AND GEOMETRIC PATTERNS

1. For each of the following sequences, write the rule for the relationship between each term and the following term in words. Then use the rule to write the next three terms in the sequence.

(a) 12; 7; 2; …….; …….; …….

……………………………………………………………

(b) –2; –6; –18; …….; …….; …….

……………………………………………………………

(c) 100; –50; 25; …….; …….; …….

……………………………………………………………

(d) 3; 4; 7; 11; …….; …….; …….

……………………………………………………………

……………………………………………………………

2. In this question, you are given the rule by which each term of the sequence can be found. In all cases, n is the position of the term.

   Determine the first three terms of each of the sequences. (*Hint*: Substitute $n = 1$ to find the value of the first term.)

   (a) $n \times 4$

   ....

   (b) $n \times 5 - 12$

   ....

   (c) $2 \times n^2$

   ....

   (d) $3n \div 3 \times -2$

   ....

3. Write down the rule by which each term of the sequence can be found (in a similar format to those given in question 2, where n is the position of the term).

   (a) 2; 4; 6; ...

   ....

   (b) –7; –3; 1; ...

   ....

   (c) 2; 4; 8; ...

   ....

   (d) 9; 16; 23; ...

   ....

4. Use the rules you have found in question 3 to find the value of the 20th term of the sequences in questions 3(a) and 3(b).

   (a) ....

   ....

   (b) ....

   ....

5. Find the relationship between the position of the term in the sequence and the value of the term, and use it to fill in the missing values in the tables.

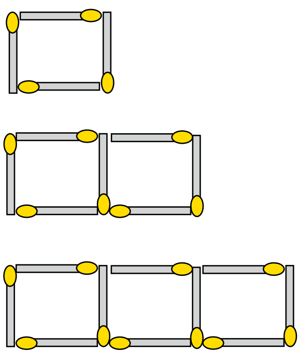
(a)

Position in sequence	1	2	3	4		25
Value of the term	–8	–11	–14			

(b)

Position in sequence	1	2	3				
Value of the term	1	3	9		243		19 683

6. The image below shows a series of patterns created by matches.



(a) Write in words the rule that describes the number of matches needed for each new pattern.

....

....

(b) Use the rule to determine the missing values in the table below, and fill them in.

Number of the pattern	1	2	3	4		20		
Number of matches needed	4	7						151

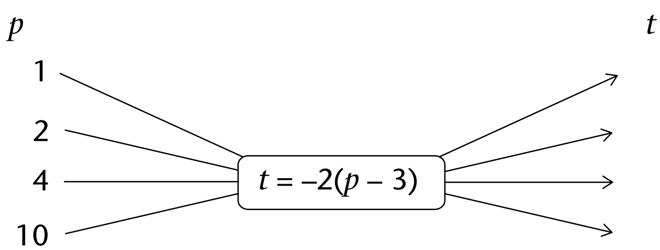
....

....

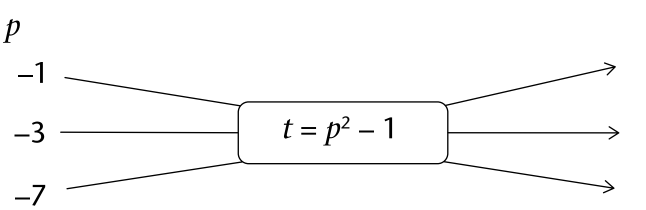
FUNCTIONS AND RELATIONSHIPS

1. Fill in the missing input values, output values or rule in these flow diagrams. Note that p and t are integers.

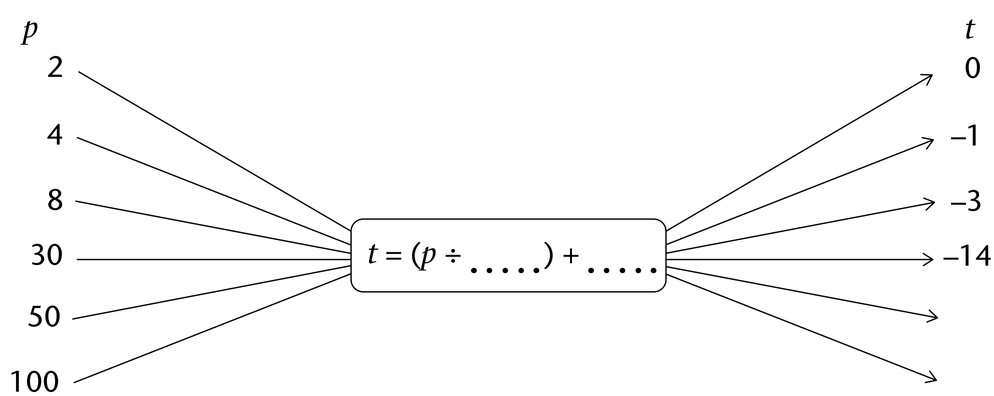
(a)



(b)



(c)



2. Consider the values in the following table. The rule for finding y is: divide x by –2 and subtract 4. Use the rule to determine the missing values in the table, and write them in.

x			–2	0	2		5		
y	–1		–3	–4					48

....

....

3. Consider the values in the following table:

x	-2	-1	0	1	2		4		15		
y	1	3	5	7	9						61

(a) Write in words the rule for finding the y-values in the table.

....

....

(b) Use the rule to determine the missing values in the table, and write them in.

....

ALGEBRAIC EXPRESSIONS 1

1. Look at this algebraic expression: $5x^3 - 9 + 4x - 3x^2$.

(a) How many terms does this expression have? ....

(b) What is the variable in this expression? ....

(c) What is the coefficient of the x^2 term? ....

(d) What is the constant in this expression? ....

(e) Rewrite the expression so that the terms are in order of decreasing powers of x.

....

2. In this question, $x = 6$ and $y = 17$. Complete the rules to show different ways to determine y if x is known. The first way is done for you:

Way 1: Multiply x by 2 and add 5. This can be written as $y = 2x + 5$

(a) Way 2: Multiply x by .... and then subtract .... This can be written as

....

(b) Way 3: Divide x by .... and then add .... This can be written as

....

(c) Way 4: Add .... to x, and then multiply by .... This can be written as

....

3. Simplify:
   (a) $2x^2 + 3x^2$ ....
   (b) $9xy - 12yx$ ....
   (c) $3y^2 - 4y + 3y - 2y^2$ ....
   (d) $9m^3 + 9m^2 + 9m^3 - 3$ ....

4. Calculate the value of the following expressions if $a = -2$; $b = 3$; $c = -1$ and $d = 0$:
   (a) abc

   ....
   (b) $-a^2$

   ....
   (c) $(abc)^d$

   ....
   (d) $a + b - 2c$

   ....
   (e) $(a + b)^{10}$

   ....

ALGEBRAIC EQUATIONS 1

1. Write equations that represent the given information:
   (a) Nandi is x years old. Shaba, who is y years old, is three years older than Nandi.

   ....
   (b) The temperature at Colesberg during the day was x °C. But at night, the temperature dropped by 15 degrees to reach -2 °C.

   ....

2. Solve the following equations for x:
   (a) $x + 5 = 2$ (b) $7 - x = 9$

   ....

   ....

(c) $3x - 1 = -10$

....

....

(d) $2x^3 = -16$

....

....

(e) $2^x = 16$

....

....

(f) $2(3)^x = 6$

....

....

3. If $3n - 1 = 11$, what is the value of $4n$?

....

....

....

4. If $c = a + b$ and $a + b + c = 16$, determine the value of c.

....

....

....

5. (a) If $2a + 3 = b$, write down values for a and b that will make the equation true.

....

(b) Write down a *different* pair of values to make the equation true.

....

Assessment

In this section, the numbers in brackets at the end of a question indicate the number of marks the question is worth. Use this information to help you determine how much working is needed. The total number of marks allocated to the assessment is 60.

1. The profits of GetRich Inc. have decreased in the ratio 5 : 3 due to the recession in the country. If their profits used to be R1 250 000, how much are their profits now? (2)

   ....

   ....

2. Which car has the better rate of petrol consumption: Ashley's car, which drove 520 km on 32 ℓ of petrol, or Zaza's car, which drove 880 km on 55 ℓ of petrol? Show all your working. (3)

   ....

   ....

   ....

3. Hanyani took out a R25 000 loan from a lender that charges him 22% interest each year. How much will he owe in one year's time? (3)

   ....

   ....

4. Consider the following exchange rates table:

South African Rand	1.00 ZAR	inv. 1.00 ZAR
Indian Rupee	*5.558584*	*0.179902*
Australian Dollar	*0.102281*	*9.776984*
Canadian Dollar	*0.101583*	*9.844200*
Emirati Dirham	*0.360838*	*2.771327*
Chinese Yuan Renminbi	*0.603065*	*1.658195*
Malaysian Ringgit	*0.303523*	*3.294646*

Chen returns from a business trip to Malaysia with 2 500 ringgit in his wallet. If he changes this money into rand in South Africa, how much will he receive? (2)

....

....

5. Fill <, > or = into the block to show the relationship between the number expressions:

   (a) $6 - 4 \;\square\; 4 - 6$ (1)

   (b) $2 \times -3 \;\square\; -3^2$ (1)

6. Look at the number sequence below. Fill in the next term into the block.

   $-5; 10; -20; \square$ (1)

7. Calculate the following:

   (a) $(-4)^2 - 20$ (2)

   ....

   (b) $\sqrt[3]{-8} + 14 \div 2$ (2)

   ....

8. Julius Caesar was a Roman emperor who lived from 100 BC to 44 BC. How old was he when he died? (2)

   ....

9. (a) Write down two numbers that divide to give an answer of –8. One of the numbers must be positive, and the other negative. (1)

   ....

   (b) Write down two numbers that subtract to give an answer of 8. One of the numbers must be positive and the other negative. (1)

   ....

10. Write the following number in scientific notation: 17 million. (2)

   ....

11. Which of the following numbers is bigger: $3{,}47 \times 10^{21}$ or $7{,}99 \times 10^{20}$? (1)

....

....

....

12. Simplify the following, leaving all answers with positive exponents:

(a) $3^7 \times 3^{-2}$ .... (1)

(b) $(-12y^8) \div (-3y^2)$ .... (2)

(c) $\dfrac{(3xy^2z^3)(-yz)^2}{15x^5y^4z^7}$ .... (4)

13. Write down the values of each of the following:

(a) $(0{,}3)^3$ .... (1)

(b) $8\sqrt{\dfrac{25}{16}}$ .... (2)

14. Consider the following number sequence: 2; –8; 32; ...

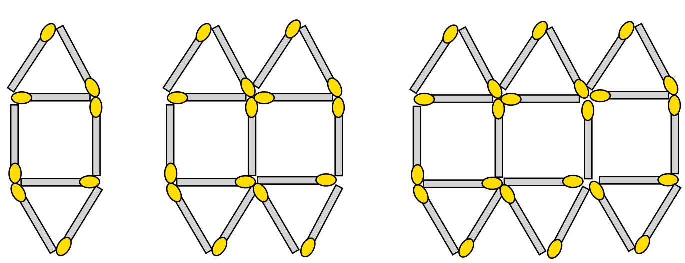
(a) Write in words the rule by which each term of the sequence can be found. (1)

....

(b) Write the next three terms in this sequence. (2)

....

15. The picture below shows a series of patterns created by matches.



(a) Write a formula for the rule that describes the relationship between the number of matches and the position of the term in the sequence (pattern number). Let n be the position of the term. (2)

....

(b) Use the rule to determine the values of a to c in the following table: (3)

Number of the pattern	1	2	3	4		15		c
Number of matches needed	8	15	22	a		b		148

....

....

....

16. Consider the values in the following table:

x	-2	-1	0	1	2		5		12	
y	-7	-4	-1	2	5					98

(a) Write in words the rule for finding the y-values in the table. (2)

....

....

(b) Use the rule to determine the missing values in the table, and fill them in. (3)

....

....

....

17. Simplify:

(a) $2z^2 - 3z^2$ .... (1)

(b) $8y^2 - 6y + 4y - 7y^2$ .... (2)

18. Determine the value of $2a^2 - 10$ if $a = -2$. (2)

....

....

....

19. If $c + 2d = 27$, give the value of the following:

(a) $2c + 4d$ .... (1)

(b) $\frac{c+2d}{-9}$ .... (1)

(c) $\sqrt[3]{c+2d}$ .... (1)

20. Solve the following for x: (5)

(a) $-x = -11$ (b) $2x - 5 = -11$ (c) $4x^3 = 32$

....

....

....

Chapter 8
Algebraic expressions 2

In this chapter, you will learn about simplifying algebraic expressions by expanding them. Expanding an algebraic expression allows you to change the form of an expression without changing the output values it gives.

Rewriting an expression in a different form can be useful for simplifying calculations and comparing expressions. We use two main tools to simplify expressions: we combine like terms and/or use the distributive property.

$$5 \times 9^2 + 4 \times 9 - 3$$

$$5 \times 8^2 + 4 \times 8 - 3$$

$$5 \times 12^2 + 4 \times 12 - 3$$

$$5 \times 20^2 + 4 \times 20 - 3$$

$$5 \times 2^2 + 4 \times 2 - 3$$

$$5 \times x^2 + 4 \times x - 3$$

$$5x^2 + 4x - 3$$

$$5 \times 7^2 + 4 \times 7 - 3$$

$$5 \times 25^2 + 4 \times 25 - 3$$

8 Algebraic expressions 2

8.1 Expanding algebraic expressions

MULTIPLY OFTEN OR MULTIPLY ONCE: IT IS YOUR CHOICE

1. (a) Calculate 5×13 and 5×87 and add the two answers.

   ....

   (b) Add 13 and 87, and then multiply the answer by 5.

   ....

   (c) If you do not get the same answer for questions 1(a) and 1(b), you have made a mistake. Redo your work until you get it right.

The fact that, if you work correctly, you get the same answer for questions 1(a) and 1(b) is an example of a certain property of addition and multiplication called the **distributive property**. You use this property each time you multiply a number in parts. For example, you may calculate 3×24 by calculating

3×20 and 3×4, and then add the two answers:

$3 \times 24 = 3 \times 20 + 3 \times 4$

What you saw in question 1 was that $5 \times 100 = 5 \times 13 + 5 \times 87$.

This can also be expressed by writing $5(13 + 87)$.

The word *distribute* means "to spread out". The distributive properties may be described as follows:
$a(b + c) = ab + ac$ and
$a(b - c) = ab - ac$,
where a, b and c can be any numbers.

2. (a) Calculate 10×56.

   ....

   (b) Calculate $10 \times 16 + 10 \times 40$.

   ....

3. Write down any two numbers smaller than 100. Let us call them x and y.

   (a) Add your two numbers, and multiply the answer by 6.

   ....

   (b) Calculate $6 \times x$ and $6 \times y$ and add the two answers.

   ....

   (c) If you do not get the same answers for (a) and (b) you have made a mistake somewhere. Correct your work.

4. Complete the table.

(a)

x	1	2	3	4	5
$3(x + 2)$					
$3x + 6$					
$3x + 2$					
$3(x - 2)$					
$3x - 6$					
$3x - 2$					

(b) If you do not get the same answers for the expressions $3(x + 2)$ and $3x + 6$, and for $3(x - 2)$ and $3x - 6$, you have made a mistake somewhere. Correct your work.

In algebra we normally write $3(x + 2)$ instead of $3 \times (x + 2)$. The expression $3 \times (x + 2)$ does not mean that you should first multiply by 3 when you evaluate the expression for a certain value of x. The brackets tell you that the first thing you should do is add the value(s) of x to 2 and then multiply the answer by 3.

However, instead of first adding the values within the brackets and then multiplying the answer by 3 we may just do the calculation $3 \times x + 3 \times 2 = 3x + 6$ as shown in the table.

(c) Which expressions amongst those given in the table are equivalent? Explain.

....

....

(d) For what value(s) of x is $3(x + 2) = 3x + 2$?

....

(e) Try to find a value of x such that $3(x + 2) \neq 3x + 6$.

....

....

....

If multiplication is the last step in evaluating an algebraic expression, then the expression is called a **product expression** or, briefly, a **product**. The way you evaluated the expression $3(x + 2)$ in the table is an example of a product expression.

5. (a) Determine the value of $5x + 15$ if $x = 6$. ....

(b) Determine the value of $5(x + 3)$ if $x = 6$. ....

(c) Can we use the expression $5x + 15$ to calculate the value of $5(x + 3)$ for any values of x? Explain.

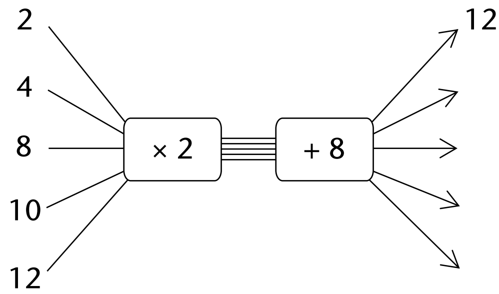
....

....

....

6. Complete the flow diagrams.

(a)



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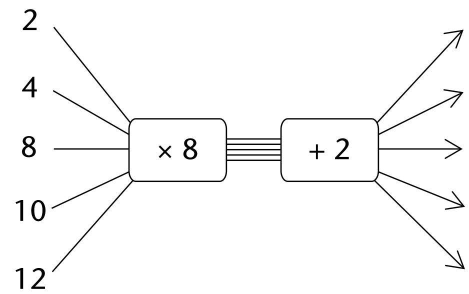

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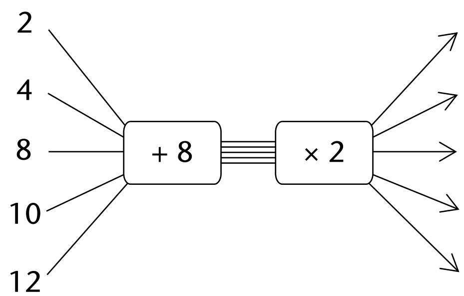

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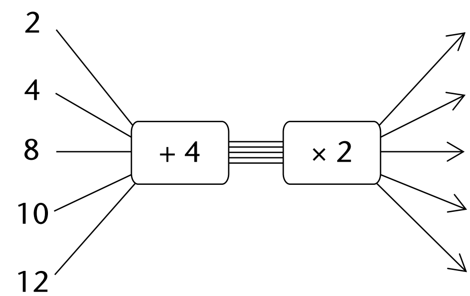

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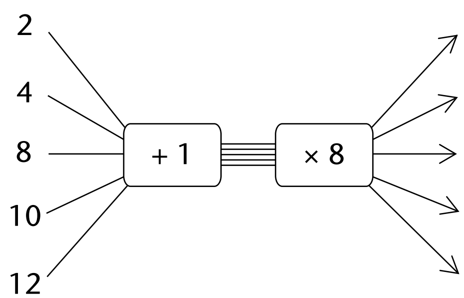

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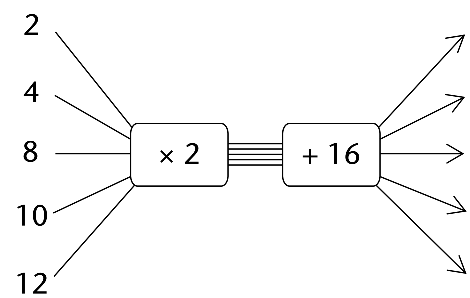


7. (a) Which of the above flow diagrams produce the same output numbers?

   ....

   (b) Write an algebraic expression for each of the flow diagrams in question 6.

   ....

   ....

   ....

PRODUCT EXPRESSIONS AND SUM EXPRESSIONS

1. Complete the following:
   (a) $(3 + 6) + (3 + 6) + (3 + 6) + (3 + 6) + (3 + 6)$

   $= \ldots\ldots \times (\ldots\ldots\ldots)$

   (b) $(3 + 6) + (3 + 6) + (3 + 6) + (3 + 6) + (3 + 6)$

   $= (3 + 3 + \ldots\ldots\ldots) + (\ldots\ldots\ldots\ldots\ldots)$

   $= (\ldots \times \ldots) + (\ldots \times \ldots)$

2. Complete the following:
   (a) $(3x + 6) + (3x + 6) + (3x + 6) + (3x + 6) + (3x + 6)$

   $= \ldots\ldots(\ldots\ldots\ldots)$

   (b) $(3x + 6) + (3x + 6) + (3x + 6) + (3x + 6) + (3x + 6)$

   $= (3x + 3x + \ldots\ldots\ldots\ldots) + (\ldots\ldots\ldots\ldots\ldots)$

   $= (\ldots \times \ldots) \ldots\ldots (\ldots \times \ldots)$

3. In each case, write an expression without brackets that will give the same results as the given expression.

   (a) $3(x + 7)$ (b) $10(2x + 1)$

   ....

   (c) $x(4x + 6)$ (d) $3(2p + q)$

   ....

   (e) $t(t + 9)$ (f) $x(y + z)$

   ....

   (g) $2b(b + a - 4)$ (h) $k^2(k - m)$

   ....

The process of writing product expressions as sum expressions is called **expansion**. It is sometimes also referred to as **multiplication of algebraic expressions**.

4. (a) Complete the table for the given values of x, y and z.

	$3(x + 2y + 4z)$	$3x + 6y + 12z$	$3x + 2y + 4z$
$x = 1$ $y = 2$ $z = 3$			
$x = 10$ $y = 20$ $z = 30$			
$x = 23$ $y = 60$ $z = 100$			
$x = 14$ $y = 0$ $z = 1$			
$x = 5$ $y = 9$ $z = 32$			

(b) Which sum expression and product expression are equivalent?

....

5. For each expression, write an equivalent expression without brackets.

(a) $2(x^2 + x + 1)$

....

(b) $p(q + r + s)$

....

(c) $-3(x + 2y + 3z)$

....

(d) $x(2x^2 + x + 7)$

....

(e) $6x(8 - 2x)$

....

(f) $12x(4 - x)$

....

(g) $3x(8x - 5) - 4x(6x - 5)$

....

(h) $10x(3x(8x - 5) - 4x(6x - 5))$

....

8.2 Simplifying algebraic expressions

EXPAND, REARRANGE AND THEN COMBINE LIKE TERMS

1. Write the shortest possible equivalent expression without brackets.

(a) $x + 2(x + 3)$

(b) $5(4x + 3) + 5x$

(c) $5(x + 5) + 3(2x + 1)$

(d) $(5 + x)^2$

(e) $-3(x^2 + 2x - 3) + 3(x^2 + 4x)$

(f) $x(x - 1) + x + 2$

When you are not sure whether you simplified an expression correctly, you should always check your work by evaluating the original expression and the simplified expression for some values of the variables.

2. (a) Evaluate $x(x + 2) + 5x^2 - 2x$ for $x = 10$.

(b) Evaluate $6x^2$ for $x = 10$.

(c) Can we use the expression $6x^2$ to calculate the values of the expression $x(x + 2) + 5x^2 - 2x$ for any given value of x? Explain.

This is how a sum expression for $x(x + 2) + 5x^2 - 2x$ can be made:

$$\begin{aligned} x(x + 2) + 5x^2 - 2x &= x \times x + x \times 2 + 5x^2 - 2x \\ &= x^2 + 2x + 5x^2 - 2x \\ &= x^2 + 5x^2 + 2x - 2x \qquad \text{[Rearrange and combine like terms]} \\ &= 6x^2 + 0 \\ &= 6x^2 \end{aligned}$$

3. Evaluate the following expressions for $x = -5$:

(a) $x + 2(x + 3)$

....

....

....

(b) $5(4x + 3) + 5x$

....

....

....

(c) $5(x + 5) + 3(2x + 1)$

....

....

....

(d) $(5 + x)^2$

....

....

....

(e) $-3(x^2 + 2x - 3) + 3(x^2 + 4x)$

....

....

....

(f) $x(x - 1) + x + 2$

....

....

....

4. Complete the table for the given values of x, y and z.

x	100	80	10	20	30
y	50	40	5	5	20
z	20	30	2	15	10
$x + (y - z)$					
$x - (y - z)$					
$x - y - z$					
$x - (y + z)$					
$x + y - z$					
$x - y + z$					

5. Say whether the following statements are true or false. Refer to the table in question 4. For any values of x, y and z:
   (a) $x + (y - z) = x + y - z$
   (b) $x - (y - z) = x - y - z$

   ....

6. Write the expressions without brackets. Do not simplify.
   (a) $3x - (2y + z)$
   (b) $-x + 3(y - 2z)$

   ....

We can simplify algebraic expressions by using properties of operations as shown:

$(5x + 3) - 2(x + 1)$

Hence $5x + 3 - 2x - 2$

Hence $5x - 2x + 3 - 2$

Hence $3x + 1$

$x - (y + z) = x - y - z$

Addition is both associative and commutative.

7. Write an equivalent expression without brackets for each of the following expressions and then simplify:
   (a) $22x + (13x - 5)$
   (b) $22x - (13x - 5)$

   ....

   ....

   (c) $22x - (13x + 5)$
   (d) $4x - (15 - 6x)$

   ....

   ....

8. Simplify.
   (a) $2(x^2 + 1) - x - 2$
   (b) $-3(x^2 + 2x - 3) + 3x^2$

   ....

   ....

Here are some of the techniques we have used so far to form equivalent expressions:

- Remove brackets
- Rearrange terms
- Combine like terms

8.3 Simplifying quotient expressions

FROM QUOTIENT EXPRESSIONS TO SUM EXPRESSIONS

1. Complete the table for the given values of x.

x	1	7	–3	–10
$7x^2 + 5x$				
$\frac{7x^2+5x}{x}$				
$7x + 5$				
$7x + 5x$				
$7x^2 + 5$				

2. (a) What is the value of $7x + 5$ for $x = 0$?

....

(b) What is the value of $\frac{7x^2+5x}{x}$ for $x = 0$?

....

(c) Which of the two expressions, $7x + 5$ or $\frac{7x^2+5x}{x}$, requires fewer calculations? Explain.

....

....

....

(d) Are the expressions $7x + 5$ and $\frac{7x^2 + 5x}{x}$ equivalent, $x = 0$ excluded? Explain.

....

(e) Are there any other expressions that are equivalent to $\frac{7x^2 + 5x}{x}$ from those given in the table? Explain.

....

....

If division is the last step in evaluating an algebraic expression, then the expression is called a **quotient expression** or an **algebraic fraction**.

The expression $\frac{7x^2 + 5x}{x}$ is an example of a quotient expression or algebraic fraction.

3. Complete the table for the given values of x.

x	5	10	–5	–10
$10x - 5x^2$				
$5x$				
$\frac{10x - 5x^2}{5x}$				
$2 - x$				

(a) What is the value of $2 - x$ for $x = 0$?

....

(b) What is the value of $\frac{10x - 5x^2}{5x}$ for $x = 0$?

....

(c) Are the expressions $2 - x$ and $\frac{10x - 5x^2}{5x}$ equivalent, $x = 0$ excluded? Explain.

....

....

....

(d) Which of the two expressions $2 - x$ or $\frac{10x - 5x^2}{5x}$ requires fewer calculations? Explain.

....

....

We have found that quotient expressions such as $\frac{10x - 5x^2}{5x}$ can sometimes be manipulated to give equivalent expressions such as $2 - x$.

The value of this is that these equivalent expressions require fewer calculations.

The expressions $\frac{10x - 5x^2}{5x}$ and $2 - x$ are not quite equivalent because for $x = 0$, the value of $2 - x$ can be calculated, while the first expression has no value.

However, we can say that the two expressions are equivalent if they have the same values for all values of x admissible for both expressions.

How is it possible that $\frac{7x^2 + 5x}{x} = 7x + 5$ and $\frac{10x - 5x^2}{5x} = 2 - x$ for all admissible values of x? We say $x = 0$ is not an admissible value of x because division by 0 is not allowed.

One of the methods for finding equivalent expressions for algebraic fractions is by means of division:

$$\begin{aligned}
\frac{7x^2 + 5x}{x} &= \frac{1}{x}(7x^2 + 5x) && \text{[just as } \tfrac{3}{5} = 3 \times \tfrac{1}{5}\text{]} \\
&= (\frac{1}{x} \times 7x^2) + (\frac{1}{x} \times 5x) && \text{[distributive property]} \\
&= \frac{7x^2}{x} + \frac{5x}{x} \\
&= 7x + 5 && \text{[provided } x \neq 0\text{]}
\end{aligned}$$

4. Use the method shown on the previous page to simplify each fraction below.

(a) $\dfrac{8x + 10z + 6}{2}$

(b) $\dfrac{20x^2 + 16x}{4}$

(c) $\dfrac{9x^2y + xy}{xy}$

(d) $\dfrac{21ab - 14a^2}{7a}$

Simplifying a quotient expression can sometimes lead to a result which still contains quotients, as you can see in the example below.

$$\frac{5x^2 + 3x}{x^2}$$

$$= \frac{5x^2}{x^2} + \frac{3x}{x^2}$$

$$= 5 + \frac{3}{x}$$

5. (a) Evaluate $\dfrac{5x^2 + 3x}{x^2}$ for $x = -1$.

(b) For the expression $\dfrac{5x^2 + 3x}{x^2}$ to be equivalent to $5 + \dfrac{3}{x}$ which value of x must be excluded? Why?

6. Simplify the following expressions:

(a) $\dfrac{8x^2 + 2x + 4}{2x}$

(b) $\dfrac{4n + 1}{n}$

....

....

7. Evaluate:

(a) $\dfrac{8x^2 + 2x + 4}{2x}$ for $x = 2$

(b) $\dfrac{4n + 1}{n}$ for $n = 4$

....

....

8. Simplify.

(a) $\dfrac{6x^4 - 12x^3 + 2}{2x}$

(b) $\dfrac{-6n^4 - 4n}{6n}$

....

....

9. When Natasha and Lebogang were asked to evaluate the expression $\dfrac{x^2 + 2x + 1}{x}$ for $x = 10$, they did it in different ways.

Natasha's calculation:

$10 + 2 + \dfrac{1}{10}$

$= 12\dfrac{1}{10}$

Lebogang's calculation:

$\dfrac{100 + 20 + 1}{10}$

$= \dfrac{121}{10}$

$= 12\dfrac{1}{10}$

Explain how each of them thought about evaluating the given expression.

....

....

....

8.4 Squares, cubes and roots of expressions

SIMPLIFYING SQUARES AND CUBES

Study the following example:

$(3x)^2 = 3x \times 3x$	Meaning of squaring
$= 3 \times x \times 3 \times x$	
$= 3 \times 3 \times x \times x$	Multiplication is commutative: $a \times b = b \times a$
$= 9x^2$	We say that $(3x)^2$ simplifies to $9x^2$

1. Simplify the expressions.
   (a) $(2x)^2$ (b) $(2x^2)^2$ (c) $(-3y)^2$

   ....

   ....

   ....

   ....

2. Simplify the expressions.
   (a) $25x - 16x$ (b) $4y + y + 3y$ (c) $a + 17a - 3a$

   ....

3. Simplify.
   (a) $(25x - 16x)^2$ (b) $(4y + y + 3y)^2$ (c) $(a + 17a - 3a)^2$

   ....

   ....

   ....

   ....

Study the following example:

$(3x)^3 = 3x \times 3x \times 3x$	Meaning of cubing
$= 3 \times x \times 3 \times x \times 3 \times x$	
$= 3 \times 3 \times 3 \times x \times x \times x$	Multiplication is commutative: $a \times b = b \times a$
$= 27x^3$	We say that $(3x)^3$ simplifies to $27x^3$

4. Simplify the following:

(a) $(2x)^3$

(b) $(-x)^3$

....

....

....

....

(c) $(5a)^3$

(d) $(7y^2)^3$

....

....

....

....

(e) $(-3m)^3$

(f) $(2x^3)^3$

....

....

....

....

5. Simplify.

(a) $5a - 2a$

(b) $7x + 3x$

(c) $4b + b$

....

6. Simplify.

(a) $(5a - 2a)^3$

(b) $(7x + 3x)^3$

(c) $(4b + b)^3$

....

....

(d) $(13x - 6x)^3$

(e) $(17x + 3x)^3$

(f) $(20y - 14y)^3$

....

....

Always remember to test whether the simplified expression is equivalent to the given expression for at least three different values of the given variable.

SQUARE AND CUBE ROOTS OF EXPRESSIONS

1. Thabang and his friend Vuyiswa were asked to simplify $\sqrt{2a^2 \times 2a^2}$.

   Thabang reasoned as follows:

   To find the square root of a number is the same as asking yourself the question: "Which number was multiplied by itself?" The number that is multiplied by itself is $2a^2$ and therefore $\sqrt{2a^2 \times 2a^2} = 2a^2$

   Vuyiswa reasoned as follows:

   I should first simplify $2a^2 \times 2a^2$ to get $4a^4$ and then calculate $\sqrt{4a^4} = 2a^2$

   Which of the two methods do you prefer? Explain why.

   ....

2. Say whether each of the following is true or false. Give a reason for your answer.

   (a) $\sqrt{6x \times 6x} = 6x$

   ....

   ....

   (b) $\sqrt{5x^2 \times 5x^2} = 5x^2$

   ....

   ....

3. Simplify.

   (a) $y^6 \times y^6$

   ....

   (b) $125x^2 + 44x^2$

   ....

4. Simplify.

   (a) $\sqrt{y^{12}}$

   ....

   ....

   ....

   (b) $\sqrt{125x^2 + 44x^2}$

   ....

   ....

   ....

   (c) $\sqrt{25a^2 - 16a^2}$

   ....

   ....

   (d) $\sqrt{121y^2}$

   ....

   ....

   (e) $\sqrt{16a^2 + 9a^2}$

   ....

   ....

   (f) $\sqrt{25a^2 - 9a^2}$

   ....

   ....

5. What does it mean to find the cube root of $8x^3$ written as $\sqrt[3]{8x^3}$?

....

6. Simplify the following:

(a) $2a \times 2a \times 2a$

....

....

(b) $10b^3 \times 10b^3 \times 10b^3$

....

....

(c) $3x^3 \times 3x^3 \times 3x^3$

....

....

(d) $-3x^3 \times -3x^3 \times -3x^3$

....

....

7. Determine the following:

(a) $\sqrt[3]{1000b^9}$

....

(b) $\sqrt[3]{2a \times 2a \times 2a}$

....

(c) $\sqrt[3]{27x^3}$

....

....

(d) $\sqrt[3]{-27x^3}$

....

....

8. Simplify the following expressions:

(a) $6x^3 + 2x^3$

....

(b) $-m^3 - 3m^3 - 4m^3$

....

9. Determine the following:

(a) $\sqrt[3]{6x^3 + 2x^3}$

....

....

....

(b) $\sqrt[3]{-8m^3}$

....

....

....

(c) $\sqrt[3]{125y^3}$

....

....

....

(d) $\sqrt[3]{93a^3 + 123a^3}$

....

....

....

WORKSHEET

1. Simplify the following:

   (a) $2(3b + 1) + 4$

   (b) $6 - (2 + 5e)$

   (c) $18mn + 22mn + 70mn$

   (d) $4pqr + 3 + 9pqr$

2. Evaluate each of the following expressions for $m = 10$:

   (a) $3m^2 + m + 10$

   (b) $5(m^2 - 5) + m^2 + 25$

3. (a) Simplify: $\dfrac{4b+6}{2}$

   (b) Evaluate the expression $\dfrac{4b+6}{2}$ for $b = 100$.

4. Simplify.

   (a) $(4g)^2$

   (b) $(6y)^3$

   (c) $(7s + 3s)^2$

5. Determine the following:

   (a) $\sqrt{121b^2}$

   (b) $\sqrt[3]{64y^3}$

   (c) $\sqrt{63d^2 + 18d^2}$

Chapter 9
Algebraic equations 2

In this chapter you will solve equations by applying inverse operations. You will also solve equations that contain exponents.

$$5 \times 9^2 + 4 \times 9 - 3$$

$$5 \times 8^2 + 4 \times 8 - 3$$

$$5 \times 12^2 + 4 \times 12 - 3$$

$$5 \times 20^2 + 4 \times 20 - 3$$

$$5 \times 2^2 + 4 \times 2 - 3$$

$$5 \times x^2 + 4 \times x - 3$$

$$5x^2 + 4x - 3$$

$$5 \times 7^2 + 4 \times 7 - 3$$

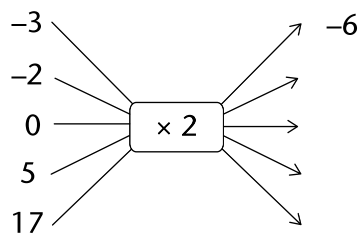
$$5 \times 25^2 + 4 \times 25 - 3$$

9 Algebraic equations 2

9.1 Thinking forwards and backwards

DOING AND UNDOING WHAT HAS BEEN DONE

1. Complete the flow diagram by finding the output values.



2. Complete the table.

x	–3	–2	0	5	17
$2x$					

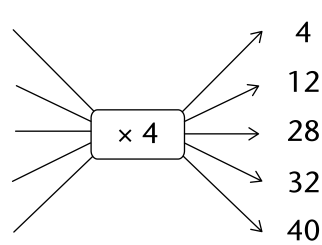
3. Evaluate $4x$ if:

(a) $x = -7$ (b) $x = 10$ (c) $x = 0$

....

....

4. (a) Complete the flow diagram by finding the input values.



(b) Puleng put another integer into the flow diagram and got –68 as an answer. Which integer did she put in? Show your calculation.

....

(c) Explain how you worked to find the input numbers when you did question (a).

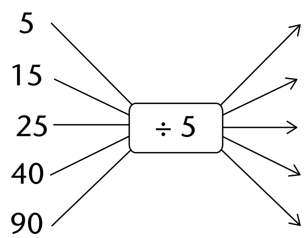
....

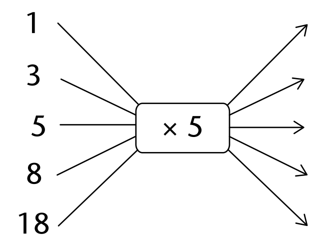
....

5. (a) Complete the table.

x					
$5x$	5	15	25	40	90

(b) Complete the flow diagrams.





(c) Explain how you completed the table.

....

One of the things we do in algebra is to **evaluate** expressions. When we evaluate expressions we replace a variable in the expression with an **input number** to obtain the value of the expression called the **output number**. We will think of this process as a **doing process**.

However, in other cases we may need to undo what was done. When we know what output number was obtained but do not know what input number was used, we have to **undo** what was done in evaluating the expression. In such a case we say we are **solving an equation.**

6. Look again at questions 1 to 5. For each question, say whether the question required a doing or an undoing process. Give an explanation for your answer (for example: input to output).

....

....

....

....

....

....

7. (a) Complete the flow diagrams below.

$1 \longrightarrow \boxed{+\ 6} \longrightarrow$ $\qquad$ $7 \longrightarrow \boxed{-\ 6} \longrightarrow$

(b) What do you observe?

....

8. (a) Complete the flow diagrams below.

$2 \longrightarrow \boxed{\times\ 5} \longrightarrow$ $\qquad$ $10 \longrightarrow \boxed{\div\ 5} \longrightarrow$

(b) What do you observe?

....

9. (a) Complete the flow diagrams below.

$20 \longrightarrow \boxed{\times\ 5} \longrightarrow \boxed{+\ 5} \longrightarrow$ $\qquad$ $105 \longrightarrow \boxed{-\ 5} \longrightarrow \boxed{\div\ 5} \longrightarrow$

(b) What do you observe?

....

10. (a) Complete the flow diagram below.

$64 \longrightarrow \boxed{\div\ 8} \longrightarrow \boxed{+\ 12} \longrightarrow$

(b) What calculations will you do to determine what the input number was when the output number is 20?

....

Solve the following problems by undoing what was done to get the answer:

11. When a certain number is multiplied by 10 the answer is 150. What is the number?

....

12. When a certain number is divided by 5 the answer is 1. What is the number?

....

13. When 23 is added to a certain number the answer is 107. What is the original number?

....

14. When a certain number is multiplied by 5 and 2 is subtracted from the answer, the final answer is 13. What is the original number?

....

Moving from the output value to the input value is called **solving the equation for the unknown**.

9.2 Solving equations using the additive and multiplicative inverses

FINDING THE UNKNOWN

Consider the equation $3x + 2 = 23$.

We can represent the equation $3x + 2 = 23$ in a flow diagram, where x represents an unknown number:

$$x \longrightarrow \boxed{\times 3} \longrightarrow \boxed{+ 2} \longrightarrow 23$$

When you reverse the process in the flow diagram, you start with the output number 23, then subtract 2 and then divide the answer by 3:

$$23 \longrightarrow \boxed{-2} \longrightarrow \boxed{\div 3} \longrightarrow$$

We can write all of the above reverse process as follows:

Subtract 2 from both sides of the equation:

$$3x + 2 - 2 = 23 - 2$$
$$3x = 21$$

Divide both sides by 3:

$$\frac{3x}{3} = \frac{21}{3}$$
$$x = 7$$

We say $x = 7$ is the solution of $3x + 2 = 23$ because $3 \times 7 + 2 = 23$. We say that $x = 7$ makes the equation $3x + 2 = 23$ true.

The numbers +2 and –2 are **additive inverses** of each other. When we add a number and its additive inverse we always get 0.

The numbers 3 and $\frac{1}{3}$ are **multiplicative inverses** of each other. When we multiply a number and its multiplicative inverse we always get 1, so $3 \times \frac{1}{3} = 1$

The additive and multiplicative inverses help us to isolate the unknown value or the input value.

Also remember:

- **The multiplicative property of 1:** the product of any number and 1 is that number.
- **The additive property of 0:** the sum of any number and 0 is that number.

Solve the equations below by using the additive and multiplicative inverses. Check your answers.

1. $x + 10 = 0$

....

....

....

....

....

2. $49x + 2 = 100$

....

....

....

....

....

3. $2x = 1$

....

....

....

....

....

4. $20 = 11 - 9x$

....

....

....

....

....

In some cases you need to collect like terms before you can solve the equations using additive and multiplicative inverses, as in the example below:

Example: Solve for x: $7x + 3x = 10$

$$10x = 10$$

$$\frac{10x}{10} = \frac{10}{10}$$

$$x = 1$$

$7x$ and $3x$ are like terms and can be replaced with one equivalent expression $(7 + 3)x = 10x$.

5. $4x + 6x = 20$

....

....

....

....

....

6. $5x = 40 + 3x$

....

....

....

....

....

7. $3x + 1 - x = 0$

8. $x + 20 + 4x = -55$

9.3 Solving equations involving powers

Solving an exponential equation is the same as asking the question: **To what exponent must the base be raised in order to make the equation true?**

1. Complete the table.

x	1	3	5	7
2^x				

2. Complete the table.

x		2		5
3^x	1		27	

Karina solved the equation $3^x = 27$ as follows:

$3^x = 27$

Hence $3^x = 3^3$

Hence $x = 3$

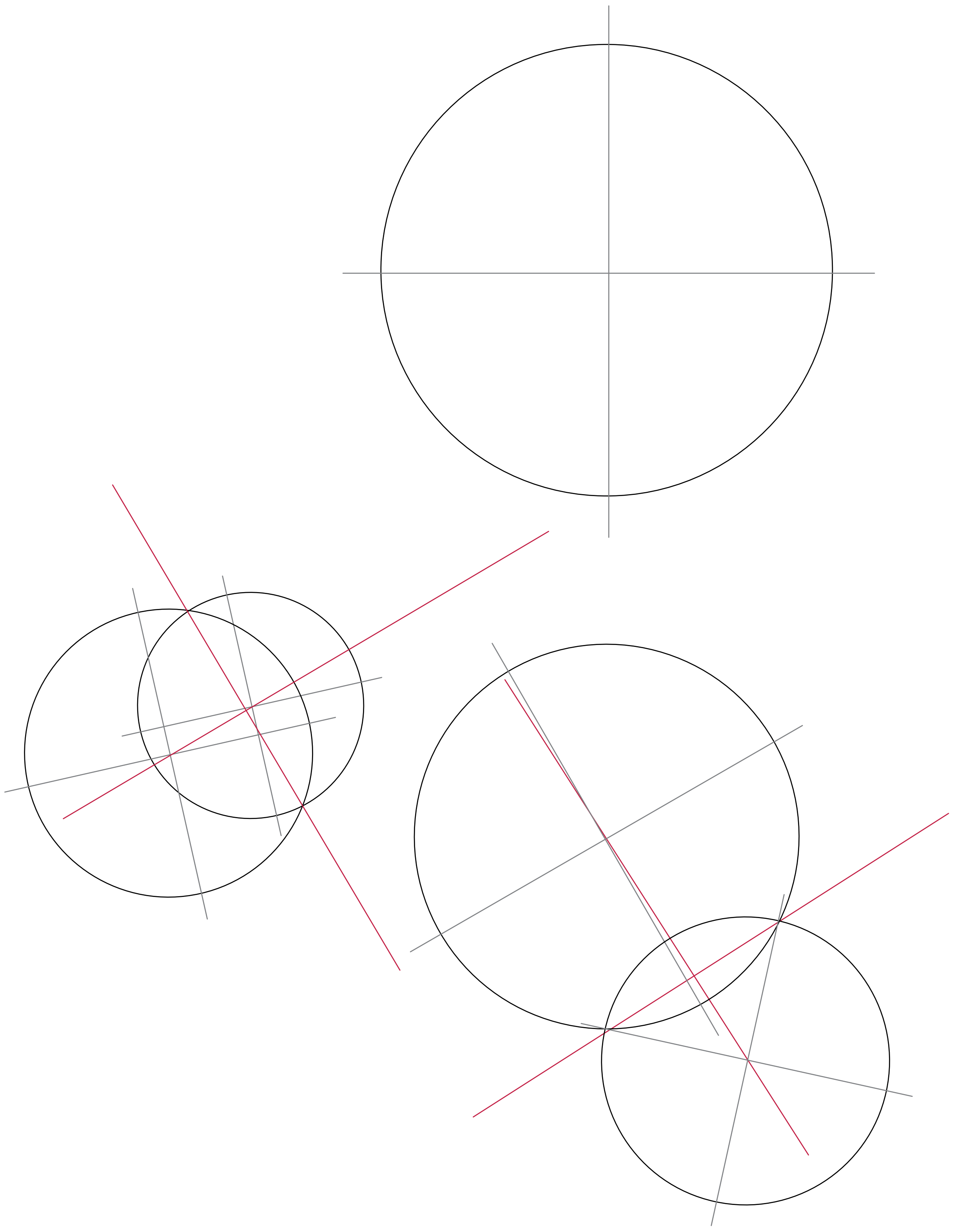
The number 27 can be expressed as 3^3 because $3^3 = 27$.

3. Now use Karina's method and solve for x in each of the following:

(a) $2^x = 32$ (b) $4^x = 16$ (c) $6^x = 216$ (d) $5^{x+1} = 125$

Chapter 10
Construction of geometric figures

In this chapter, you will learn how to construct, or draw, different lines, angles and shapes. You will use drawing instruments, such as a ruler, to draw straight lines, a protractor to measure and draw angles, and a compass to draw arcs that are a certain distance from a point. Through the various constructions, you will investigate some of the properties of triangles and quadrilaterals; in other words, you will find out more about what is always true about all or certain types of triangles and quadrilaterals.



Can two circles be drawn so that the red lines do not cross at right angles?

10 Construction of geometric figures

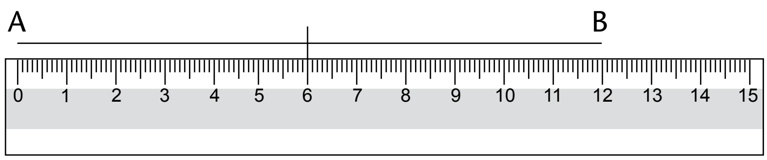
10.1 Bisecting lines

When we construct, or draw, geometric figures, we often need to bisect lines or angles. **Bisect** means to cut something into two equal parts. There are different ways to bisect a line segment.

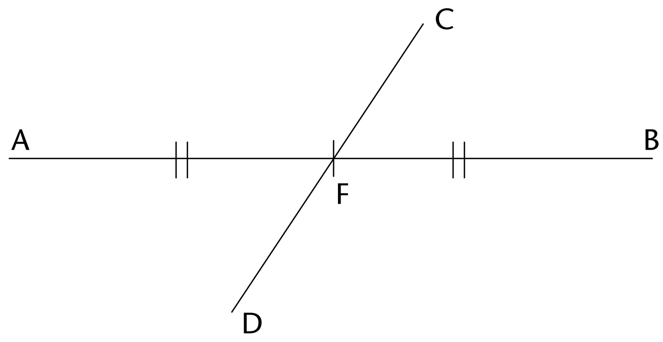
BISECTING A LINE SEGMENT WITH A RULER

1. Read through the following steps.

Step 1: Draw line segment AB and determine its midpoint.



Step 2: Draw any line segment through the midpoint.



The small marks on AF and FB show that AF and FB are equal.

CD is called a **bisector** because it bisects AB. AF = FB.

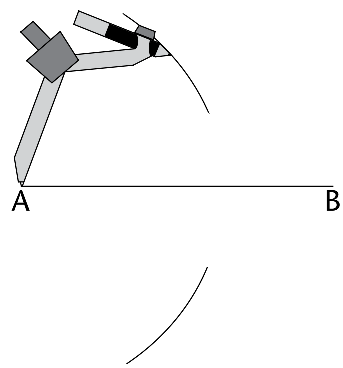
2. Use a ruler to draw and bisect the following line segments: AB = 6 cm and XY = 7 cm.

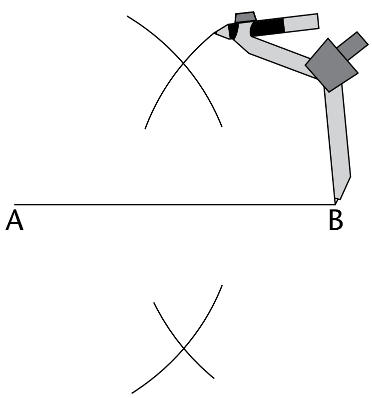
In Grade 6, you learnt how to use a compass to draw circles, and parts of circles called arcs. We can use arcs to bisect a line segment.

BISECTING A LINE SEGMENT WITH A COMPASS AND RULER

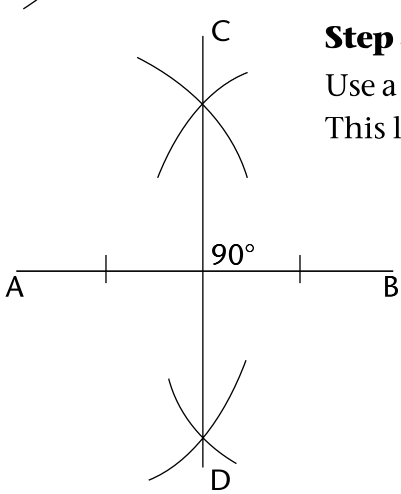
1. Read through the following steps.

Step 1
Place the compass on one endpoint of the line segment (point A). Draw an arc above and below the line. (Notice that all the points on the arc above and below the line are the same distance from point A.)





Step 2
Without changing the compass width, place the compass on point B. Draw an arc above and below the line so that the arcs cross the first two. (The two points where the arcs cross are the same distance away from point A and from point B.)



Step 3
Use a ruler to join the points where the arcs **intersect**. This line segment (CD) is the bisector of AB.

Intersect means to cross or meet.
A **perpendicular** is a line that meets another line at an angle of 90°.

Notice that CD is also **perpendicular** to AB. So it is also called a **perpendicular bisector**.

2. Work in your exercise book. Use a compass and a ruler to practise drawing perpendicular bisectors on line segments.

Try this!
Work in your exercise book. Use *only a protractor and ruler* to draw a perpendicular bisector on a line segment. (Remember that we use a protractor to measure angles.)

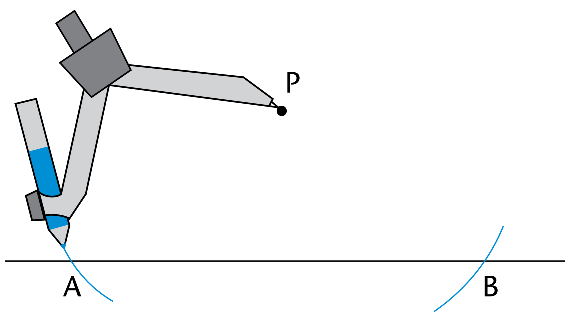
10.2 Constructing perpendicular lines

A PERPENDICULAR LINE FROM A GIVEN POINT

1. Read through the following steps.

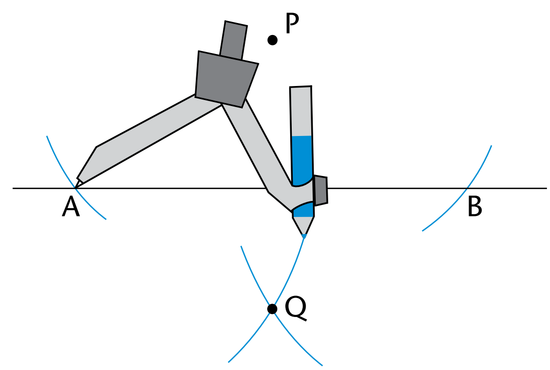
Step 1

Place your compass on the given point (point P). Draw an arc across the line on each side of the given point. Do not adjust the compass width when drawing the second arc.



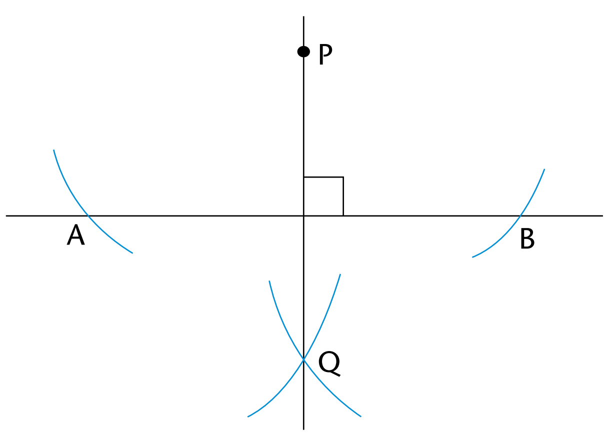
Step 2

From each arc on the line, draw another arc on the opposite side of the line from the given point (P). The two new arcs will intersect.



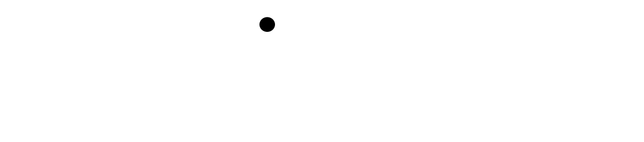
Step 3

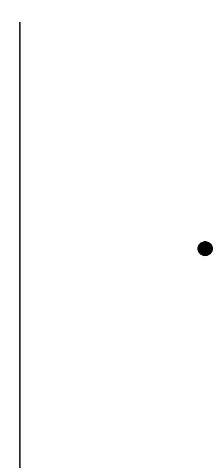
Use your ruler to join the given point (P) to the point where the arcs intersect (Q).



PQ is perpendicular to AB.
We also write it like this: $PQ \perp AB$.

2. Use your compass and ruler to draw a perpendicular line from each given point to the line segment:



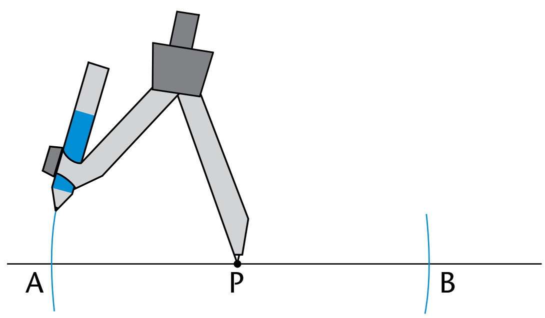


A PERPENDICULAR LINE AT A GIVEN POINT ON A LINE

1. Read through the following steps.

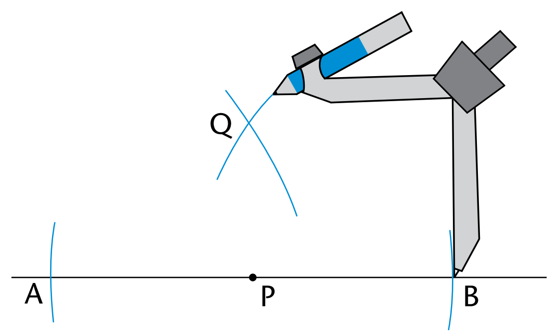
Step 1

Place your compass on the given point (P). Draw an arc across the line on each side of the given point. Do not adjust the compass width when drawing the second arc.



Step 2

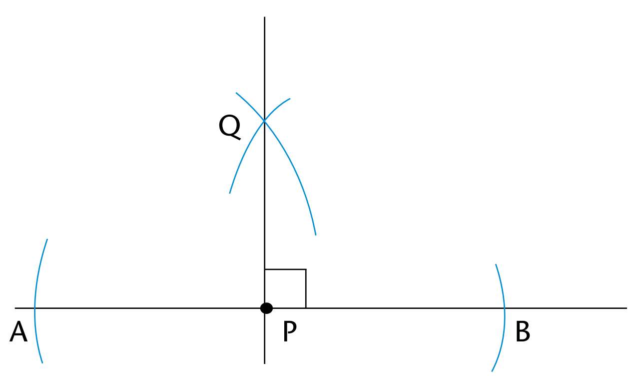
Open your compass so that it is wider than the distance from one of the arcs to the point P. Place the compass on each arc and draw an arc above or below the point P. The two new arcs will intersect.



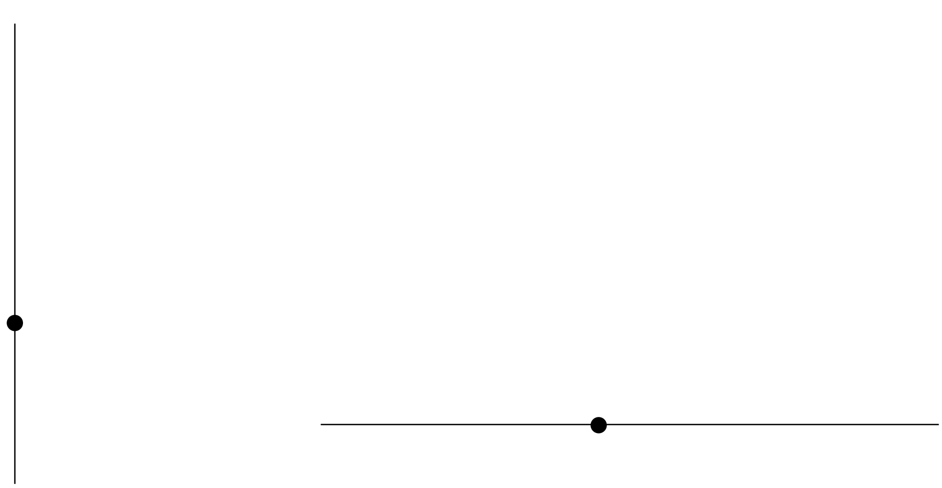
Step 3

Use your ruler to join the given point (P) and the point where the arcs intersect (Q).

$PQ \perp AB$



2. Use your compass and ruler to draw a perpendicular at the given point on each line:



10.3 Bisecting angles

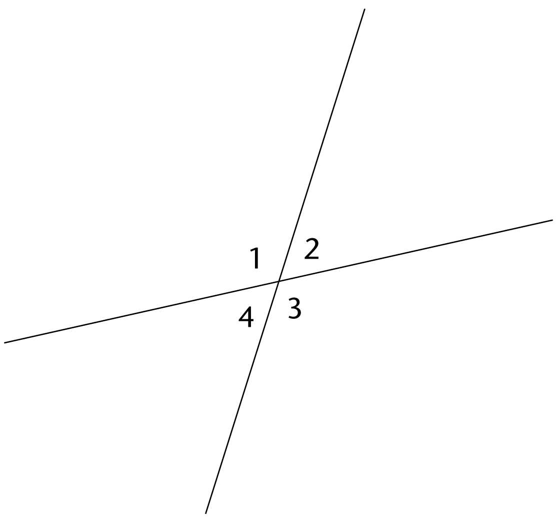
Angles are formed when any two lines meet. We use degrees (°) to measure angles.

MEASURING AND CLASSIFYING ANGLES

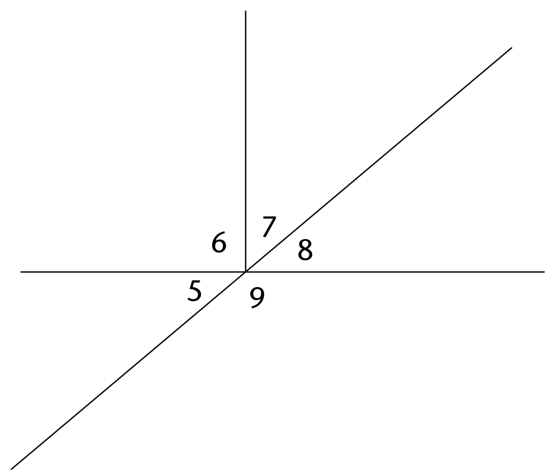
In the figures below, each angle has a number from 1 to 9.

1. Use a protractor to measure the sizes of all the angles in each figure. Write your answers on each figure.

(a)



(b)



2. Use your answers to fill in the angle sizes below.

$\hat{1} =$ ..... °	$\hat{6} =$ ..... ° ....
$\hat{1} + \hat{2} =$ ..... °	$\hat{7} + \hat{8} =$ ..... ° ....
$\hat{1} + \hat{4} =$ ..... °	$\hat{6} + \hat{7} + \hat{8} =$ ..... ° ....
$\hat{2} + \hat{3} =$ ..... °	$\hat{5} + \hat{6} + \hat{7} =$ ..... ° ....
$\hat{3} + \hat{4} =$ ..... °	$\hat{6} + \hat{5} =$ ..... ° ....
$\hat{1} + \hat{2} + \hat{4} =$ ..... °	$\hat{5} + \hat{6} + \hat{7} + \hat{8} =$ ..... ° ....
$\hat{1} + \hat{2} + \hat{3} + \hat{4} =$ ..... °	$\hat{5} + \hat{6} + \hat{7} + \hat{8} + \hat{9} =$ ..... ° ....

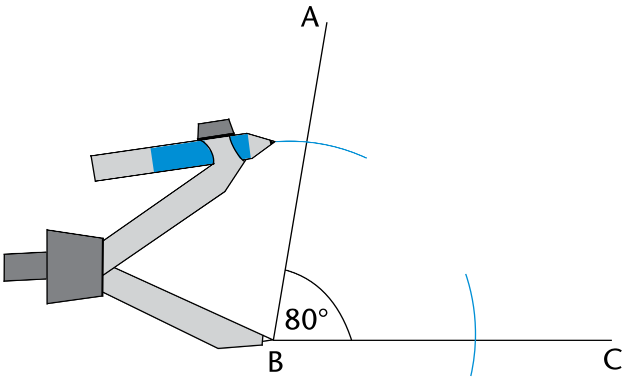
3. Next to each answer above, write down what type of angle it is, namely acute, obtuse, right, straight, reflex or a revolution.

BISECTING ANGLES WITHOUT A PROTRACTOR

1. Read through the following steps.

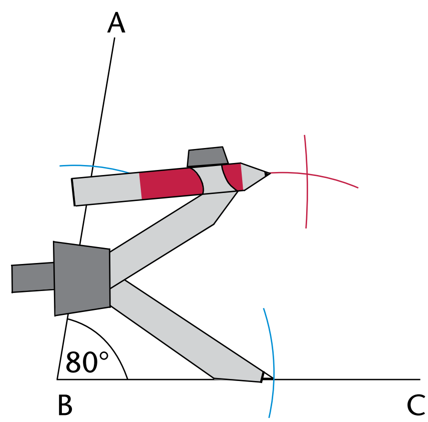
Step 1

Place the compass on the vertex of the angle (point B). Draw an arc across each arm of the angle.



Step 2

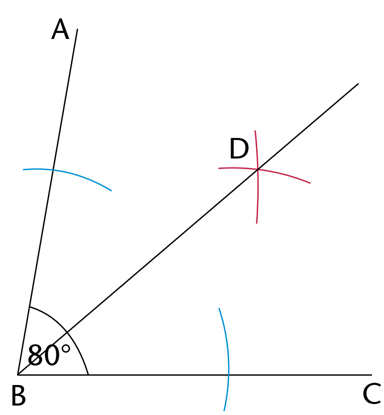
Place the compass on the point where one arc crosses an arm and draw an arc inside the angle. Without changing the compass width, repeat for the other arm so that the two arcs cross.



Step 3

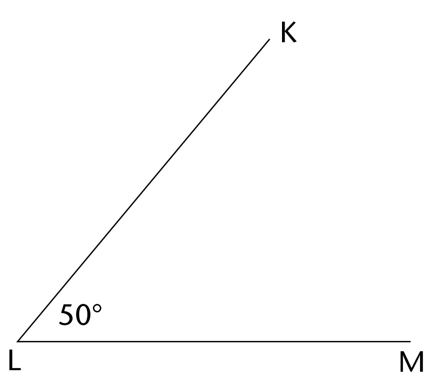
Use a ruler to join the vertex to the point where the arcs intersect (D).

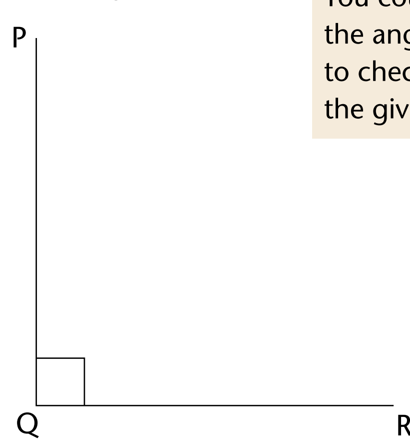
DB is the bisector of $A\hat{B}C$.



2. Use your compass and ruler to bisect the angles below.

You could measure each of the angles with a protractor to check if you have bisected the given angle correctly.





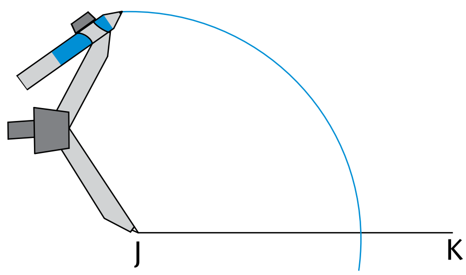
10.4 Constructing special angles without a protractor

CONSTRUCTING ANGLES OF 60°, 30° AND 120°

1. Read through the following steps.

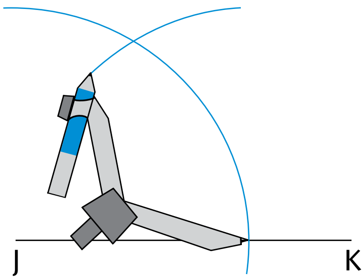
Step 1

Draw a line segment (JK). With the compass on point J, draw an arc across JK and up over above point J.



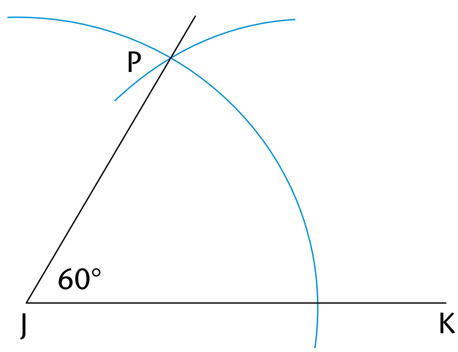
Step 2

Without changing the compass width, move the compass to the point where the arc crosses JK, and draw an arc that crosses the first one.



Step 3

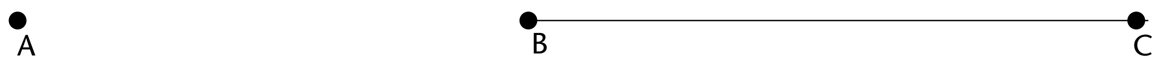
Join point J to the point where the two arcs meet (point P).
$P\hat{J}K = 60°$



2. (a) Construct an angle of 60° at point B on the next page.
   (b) Bisect the angle you constructed.
   (c) Do you notice that the bisected angle consists of two 30° angles?
   (d) Extend line segment BC to A.
   Then measure the angle adjacent to the 60° angle.
   What is its size? . . . .
   (e) The 60° angle and its adjacent angle add up to . . . .

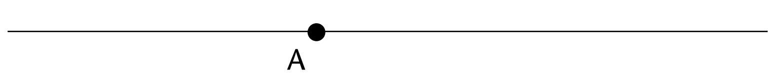
When you learn more about the properties of triangles later, you will understand why the method above creates a 60° angle. Or can you already work this out now? (*Hint*: What do you know about equilateral triangles?)

Adjacent means “next to”.



CONSTRUCTING ANGLES OF 90° AND 45°

1. Construct an angle of 90° at point A. Go back to section 10.2 if you need help.
2. Bisect the 90° angle, to create an angle of 45°. Go back to section 10.3 if you need help.



Challenge
Work in your exercise book. Try to construct the following angles without using a protractor: 150°, 210° and 135°.

10.5 Constructing triangles

In this section, you will learn how to construct triangles. You will need a pencil, a protractor, a ruler and a compass.

A triangle has three sides and three angles. We can construct a triangle when we know some of its measurements, that is, its sides, its angles, or some of its sides and angles.

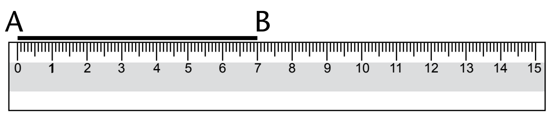
CONSTRUCTING TRIANGLES

Constructing triangles when three sides are given

1. Read through the following steps. They describe how to construct ΔABC with side lengths of 3 cm, 5 cm and 7 cm.

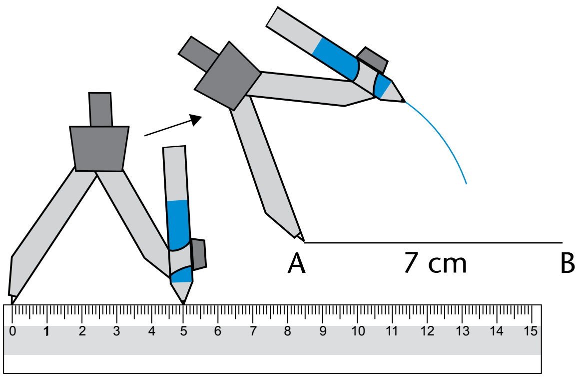
Step 1

Draw one side of the triangle using a ruler. It is often easier to start with the longest side.



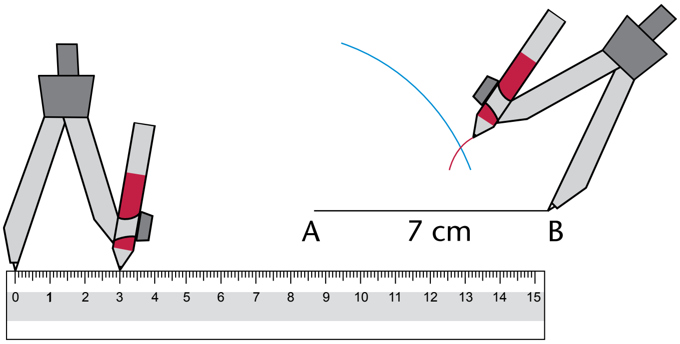
Step 2

Set the compass width to 5 cm. Draw an arc 5 cm away from point A. The third vertex of the triangle will be somewhere along this arc.



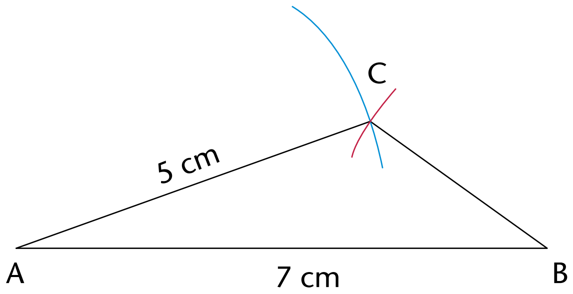
Step 3

Set the compass width to 3 cm. Draw an arc from point B. Note where this arc crosses the first arc. This will be the third vertex of the triangle.



Step 4

Use your ruler to join points A and B to the point where the arcs intersect (C).

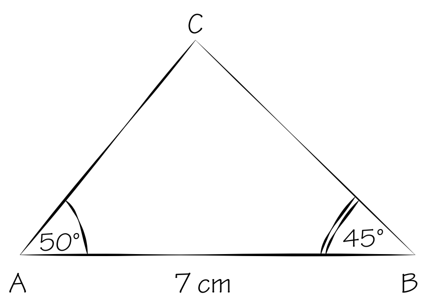


2. Work in your exercise book. Follow the steps above to construct the following triangles:
   (a) ΔABC with sides 6 cm, 7 cm and 4 cm
   (b) ΔKLM with sides 10 cm, 5 cm and 8 cm
   (c) ΔPQR with sides 5 cm, 9 cm and 11 cm

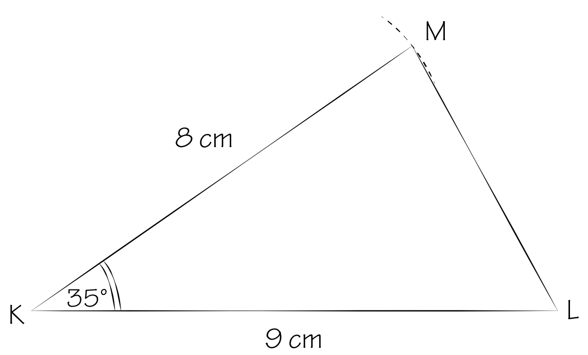
Constructing triangles when certain angles and sides are given

3. Use the rough sketches in (a) to (c) below to construct accurate triangles, using a ruler, compass and protractor. Do the construction next to each rough sketch.
   - The dotted lines show where you have to use a compass to measure the length of a side.
   - Use a protractor to measure the size of the given angles.

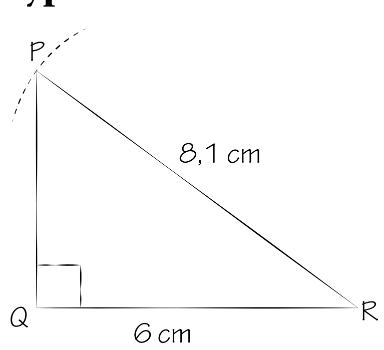
   (a) Construct ΔABC, with **two angles and one side given**.



   (b) Construct a ΔKLM, with **two sides and an angle given**.



   (c) Construct right-angled ΔPQR, with the **hypotenuse and one other side given**.



4. Measure the missing angles and sides of each triangle in 3(a) to (c) on the previous page. Write the measurements at your completed constructions.
5. Compare each of your constructed triangles in 3(a) to (c) with a classmate's triangles. Are the triangles exactly the same?

If triangles are exactly the same, we say they are **congruent**.

Challenge

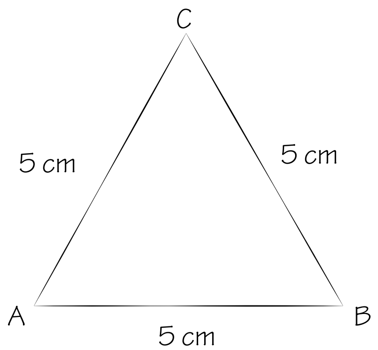
1. Construct these triangles:
   (a) ΔSTU, **with three angles given**: $\hat{S} = 45°$, $\hat{T} = 70°$ and $\hat{U} = 65°$.
   (b) ΔXYZ, with two sides and the angle opposite one of the sides given: $\hat{X} = 50°$, XY = 8 cm and XZ = 7 cm.
2. Can you find more than one solution for each triangle above? Explain your findings to a classmate.

10.6 Properties of triangles

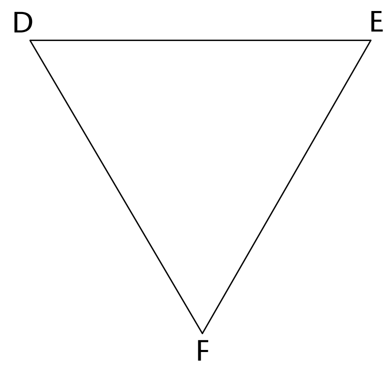
The angles of a triangle can be the same size or different sizes. The sides of a triangle can be the same length or different lengths.

PROPERTIES OF EQUILATERAL TRIANGLES

1. (a) Construct ΔABC next to its rough sketch below.
   (b) Measure and label the sizes of all its sides and angles.



2. Measure and write down the sizes of the sides and angles of ΔDEF on the right.



3. Both triangles in questions 1 and 2 are called **equilateral triangles**. Discuss with a classmate if the following is true for an equilateral triangle:
   - All the sides are equal.
   - All the angles are equal to 60°.

PROPERTIES OF ISOSCELES TRIANGLES

1. (a) Construct ΔDEF with $EF = 7$ cm, $\hat{E} = 50°$ and $\hat{F} = 50°$.
   Also construct ΔJKL with $JK = 6$ cm, $KL = 6$ cm and $\hat{J} = 70°$.

   (b) Measure and label all the sides and angles of each triangle.

2. Both triangles above are called **isosceles triangles**. Discuss with a classmate whether the following is true for an isosceles triangle:
   - Only two sides are equal.
   - Only two angles are equal.
   - The two equal angles are opposite the two equal sides.

THE SUM OF THE ANGLES IN A TRIANGLE

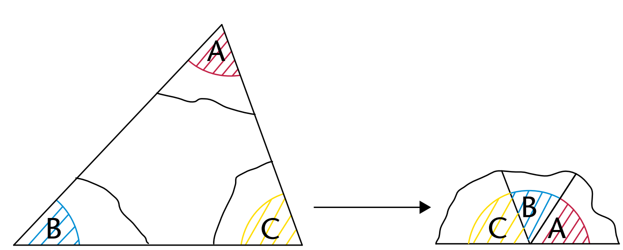
1. Look at your constructed triangles ΔABC, ΔDEF and ΔJKL above and on the previous page. What is the sum of the three angles each time? ....

2. Did you find that the sum of the interior angles of each triangle is 180°? Do the following to check if this is true for other triangles.

   (a) On a clean sheet of paper, construct any triangle. Label the angles A, B and C and cut out the triangle.

   (b) Neatly tear the angles off the triangle and fit them next to one another.

   (c) Notice that $\hat{A}$, $\hat{B}$ and $\hat{C}$ form a straight angle. Complete: $\hat{A} + \hat{B} + \hat{C} =$ ....°



We can conclude that the interior angles of a triangle always add up to 180°.

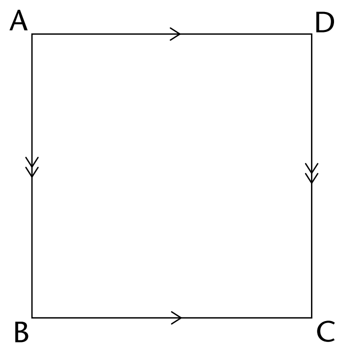
10.7 Properties of quadrilaterals

A quadrilateral is any closed shape with four straight sides. We classify quadrilaterals according to their sides and angles. We note which sides are parallel, perpendicular or equal. We also note which angles are equal.

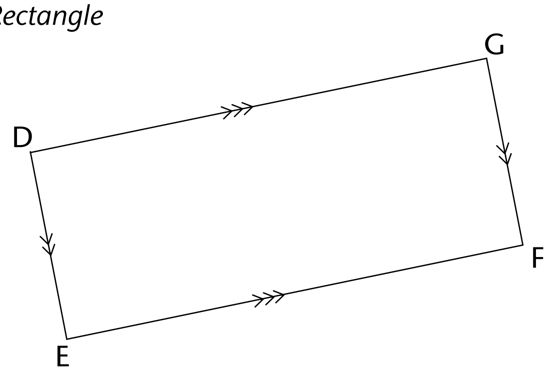
PROPERTIES OF QUADRILATERALS

1. Measure and write down the sizes of all the angles and the lengths of all the sides of each quadrilateral below.

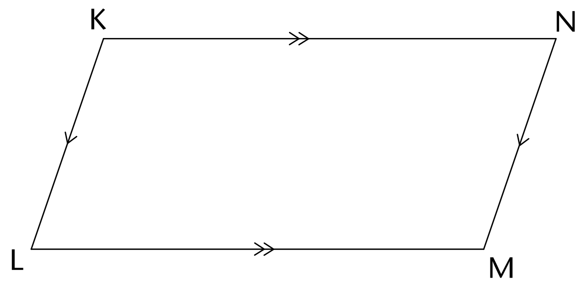
Square



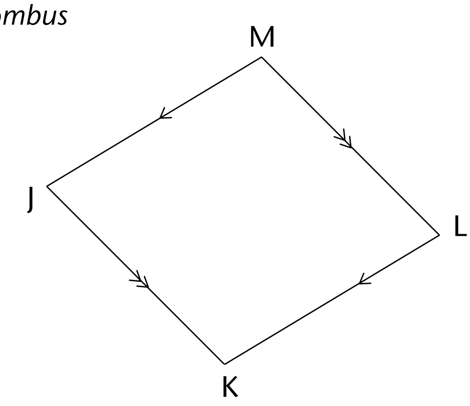
Rectangle



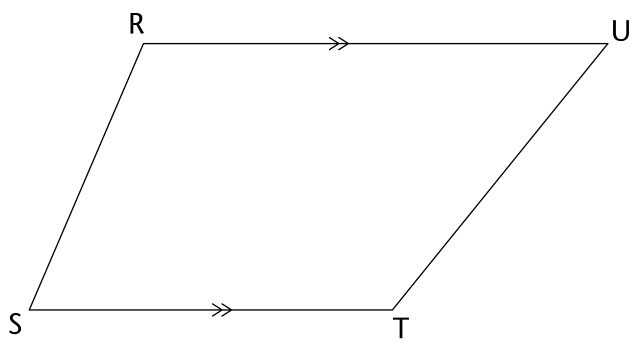
Parallelogram



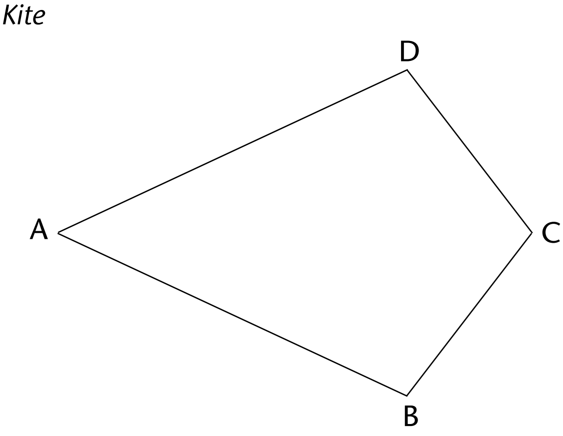
Rhombus



Trapezium



Kite



2. Use your answers in question 1. Place a ✓ in the correct box below to show which property is correct for each shape.

Properties	**Parallelogram**	**Rectangle**	**Rhombus**	**Square**	**Kite**	**Trapezium**
Only one pair of sides are parallel						
Opposite sides are parallel						
Opposite sides are equal						
All sides are equal						
Two pairs of adjacent sides are equal						
Opposite angles are equal						
All angles are equal						

SUM OF THE ANGLES IN A QUADRILATERAL

1. Add up the four angles of each quadrilateral on the previous page. What do you notice about the sum of the angles of each quadrilateral?

   ....

2. Did you find that the sum of the interior angles of each quadrilateral equals 360°? Do the following to check if this is true for other quadrilaterals.
   (a) On a clean sheet of paper, use a ruler to construct any quadrilateral.
   (b) Label the angles A, B, C and D. Cut out the quadrilateral.
   (c) Neatly tear the angles off the quadrilateral and fit them next to one another.
   (d) What do you notice?

   ....

We can conclude that the interior angles of a quadrilateral always add up to 360°.

10.8 Constructing quadrilaterals

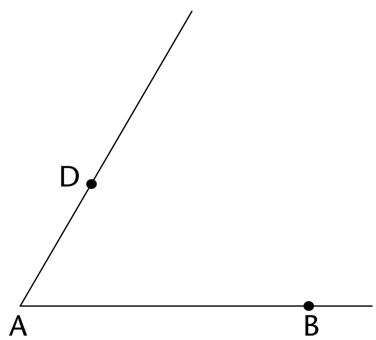
You learnt how to construct perpendicular lines in section 10.2. If you know how to construct parallel lines, you should be able to construct any quadrilateral accurately.

CONSTRUCTING PARALLEL LINES TO DRAW QUADRILATERALS

1. Read through the following steps.

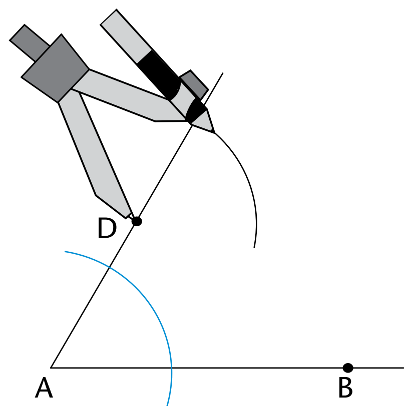
Step 1

From line segment AB, mark a point D. This point D will be on the line that will be parallel to AB. Draw a line from A through D.



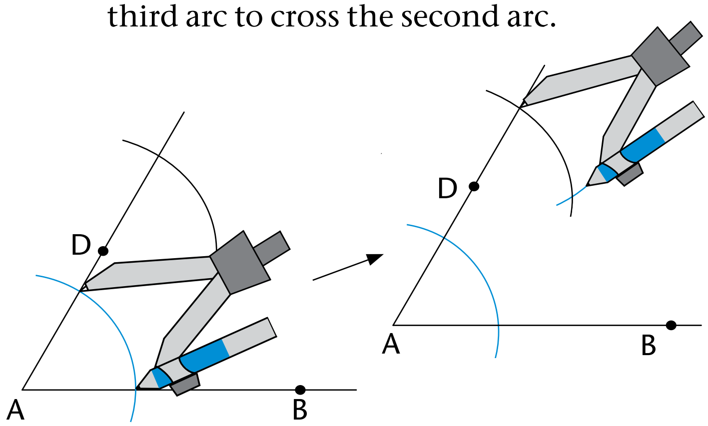
Step 2

Draw an arc from A that crosses AD and AB. Keep the same compass width and draw an arc from point D as shown.



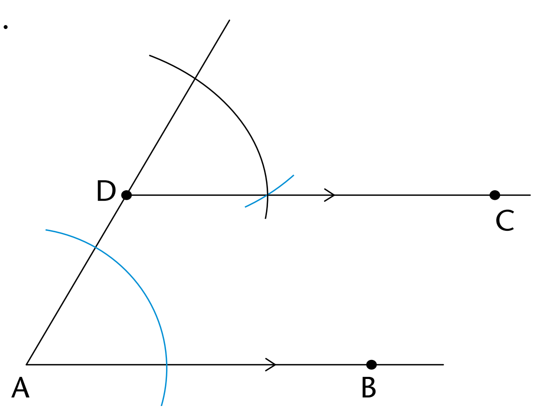
Step 3

Set the compass width to the distance between the two points where the first arc crosses AD and AB. From the point where the second arc crosses AD, draw a third arc to cross the second arc.



Step 4

Draw a line from D through the point where the two arcs meet. DC is parallel to AB.



2. Practise drawing a parallelogram, square and rhombus in your exercise book.
3. Use a protractor to try to draw quadrilaterals with at least one set of parallel lines.

WORKSHEET

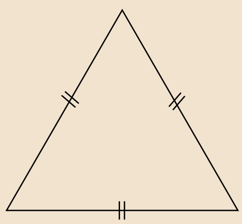
1. Do the following construction in your exercise book.

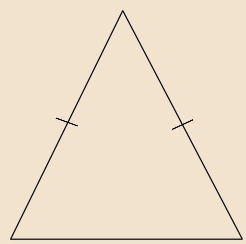
   (a) Use a compass and ruler to construct equilateral ΔABC with sides 9 cm.

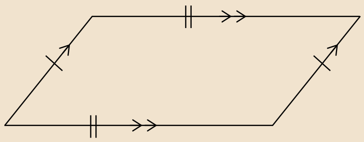
   (b) Without using a protractor, bisect $\hat{B}$. Let the bisector intersect AC at point D.

   (c) Use a protractor to measure $A\hat{D}B$. Write the measurement on the drawing.

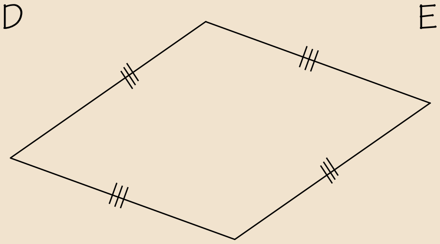
2. Name the following types of triangles and quadrilaterals.

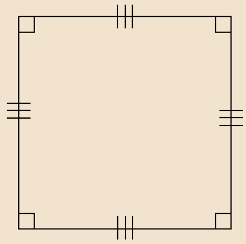
A


B


C


....

D


E


F


....

3. Which of the following quadrilaterals matches each description below? (There may be more than one answer for each.)

parallelogram; rectangle; rhombus; square; kite; trapezium

   (a) All sides are equal and all angles are equal. ....

   (b) Two pairs of adjacent sides are equal. ....

   (c) One pair of sides is parallel. ....

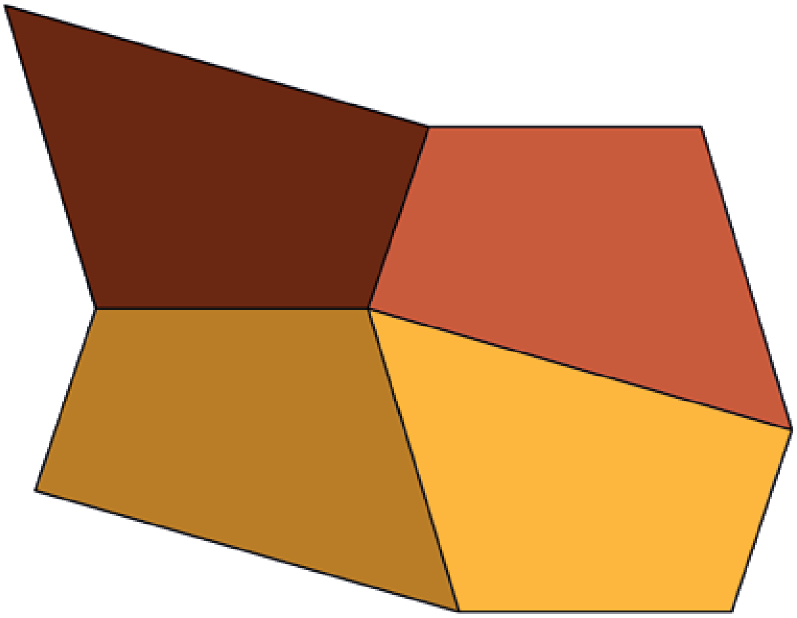
   (d) Opposite sides are parallel. ....

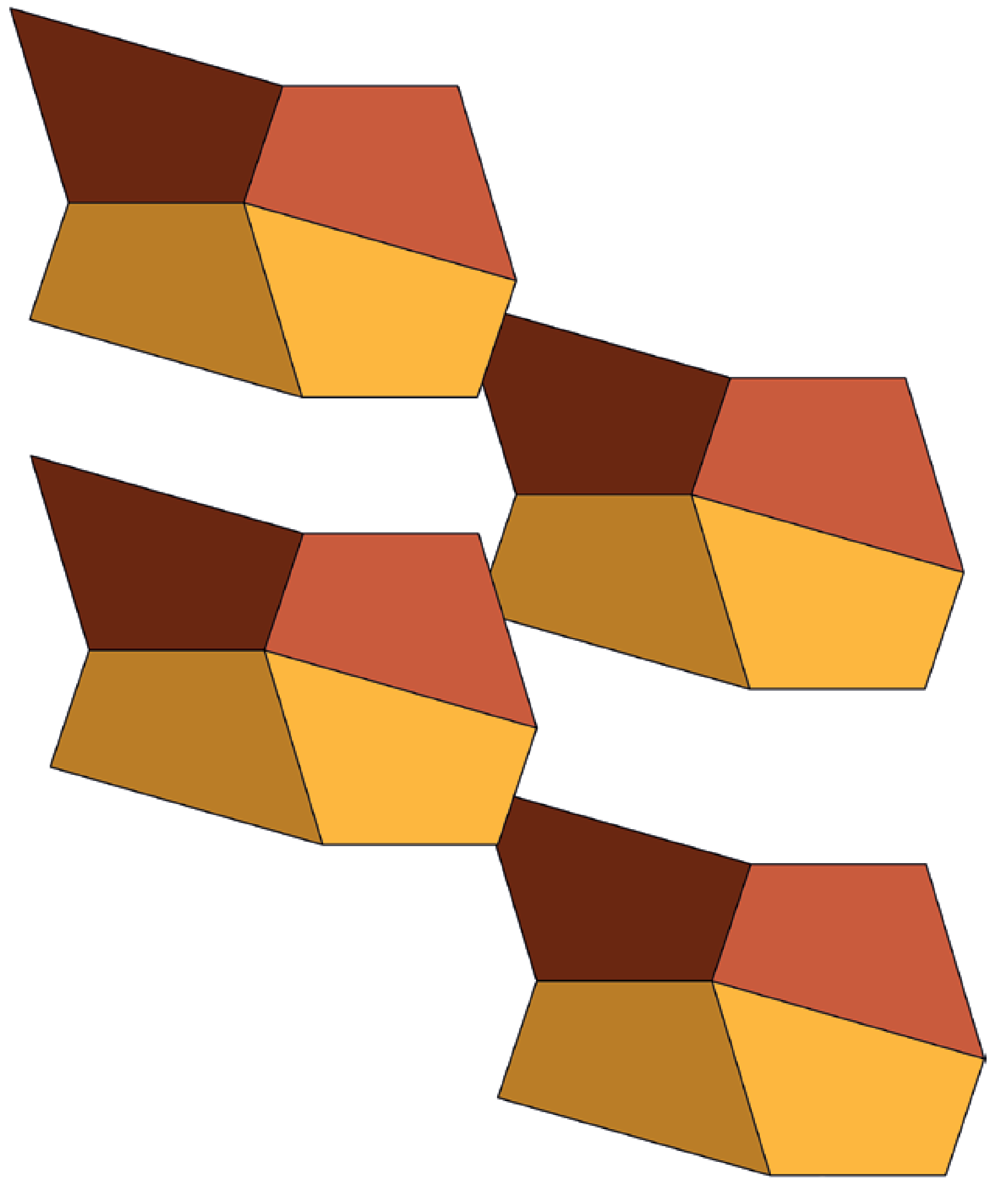
   (e) Opposite sides are parallel and all angles are equal. ....

   (f) All sides are equal. ....

Chapter 11
Geometry of 2D shapes

In this chapter, you will learn more about different kinds of triangles and quadrilaterals, and their properties. You will explore shapes that are congruent and shapes that are similar. You will also use your knowledge of the properties of 2D shapes in order to solve geometric problems.





11 Geometry of 2D shapes

11.1 Types of triangles

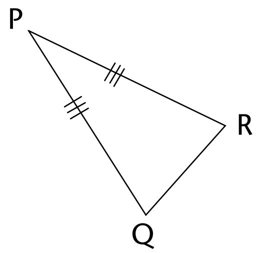
By now, you know that a triangle is a closed 2D shape with three straight sides. We can classify or name different types of triangles according to the lengths of their sides and according to the sizes of their angles.

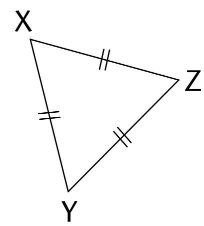
NAMING TRIANGLES ACCORDING TO THEIR SIDES

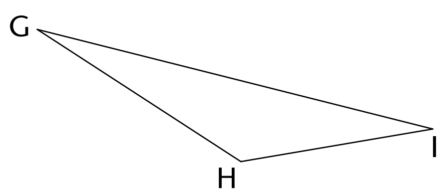
1. Match the name of each type of triangle with its correct description.

Name of triangle	Description of triangle
Isosceles triangle	All the sides of a triangle are equal.
Scalene triangle	None of the sides of a triangle are equal.
Equilateral triangle	Two sides of a triangle are equal.

2. Name each type of triangle by looking at its sides.







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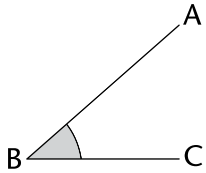
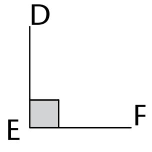
NAMING TRIANGLES ACCORDING TO THEIR ANGLES

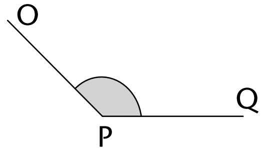
Remember the following types of angles:



Acute angle
(< 90°)

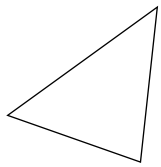


Right angle
(= 90°)

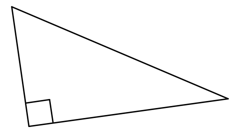


Obtuse angle
(between 90° and 180°)

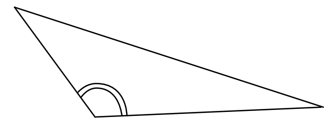
Study the following triangles; then answer the questions:



Acute triangle



Right-angled triangle



Obtuse triangle

1. Are all the angles of a triangle always equal? ....
2. When a triangle has an obtuse angle, it is called an ....triangle.
3. When a triangle has only acute angles, it is called an ....triangle.
4. When a triangle has an angle equal to ...., it is called a right-angled triangle.

INVESTIGATING THE ANGLES AND SIDES OF TRIANGLES

If you cannot work out the answers in 1(b) and (c), try to construct the triangles to find the answers.

1. (a) What is the sum of the interior angles of a triangle? ....

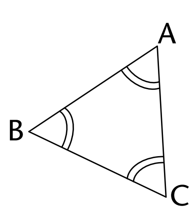
   (b) Can a triangle have two right angles? Explain your answer.

   ....

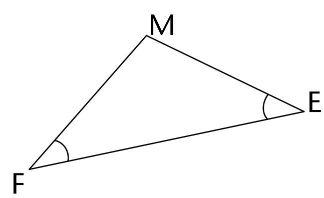
   (c) Can a triangle have more than one obtuse angle? Explain your answer.

   ....

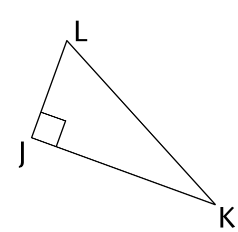
2. Look at the triangles below. The arcs show which angles are equal.



Equilateral triangle



Isosceles triangle



Right-angled triangle

   (a) ΔABC is an equilateral triangle. What do you notice about its angles?

   ....

   (b) ΔFEM is an isosceles triangle. What do you notice about its angles?

   ....

(c) ΔJKL is a right-angled triangle. Is its longest side opposite the 90° angle? ....

(d) Construct any three right-angled triangles on a sheet of paper. Is the longest side always opposite the 90° angle?

....

Properties of triangles:

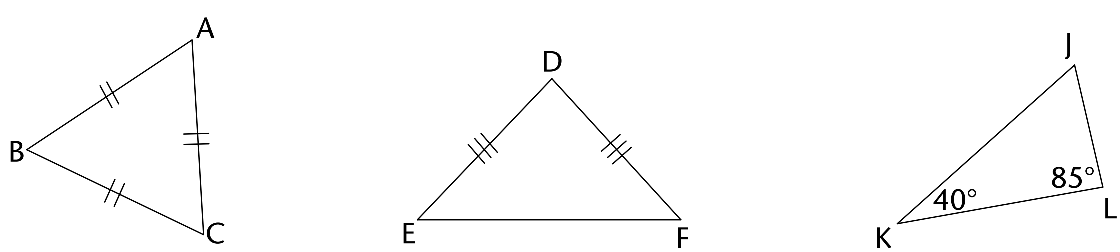
- The **sum of the interior angles** of a triangle is 180°.
- An **equilateral triangle** has all sides equal and each interior angle is equal to 60°.
- An **isosceles triangle** has two equal sides and the angles opposite the equal sides are equal.
- A **scalene triangle** has no sides equal.
- A **right-angled triangle** has a right angle (90°).
- An **obtuse triangle** has one obtuse angle (between 90° and 180°).
- An **acute triangle** has three acute angles (< 90°).

Interior angles are the angles inside a closed shape, not the angles outside of it.

11.2 Unknown angles and sides of triangles

You can use what you know about triangles to obtain other information. When you work out new information, you must always give reasons for the statements you make.

Look at the examples below of working out unknown angles and sides when certain information is given. The reason for each statement is written in square brackets.



$\hat{A} = \hat{B} = \hat{C} = 60°$	[Angles in an equilateral Δ = 60°]
DE = DF	[Given]
$\hat{E} = \hat{F}$	[Angles opposite the equal sides of an isosceles Δ are equal]
$\hat{J} = 55°$	[The sum of the interior angles of a Δ = 180°; so $\hat{J} = 180° - 40° - 85°$]

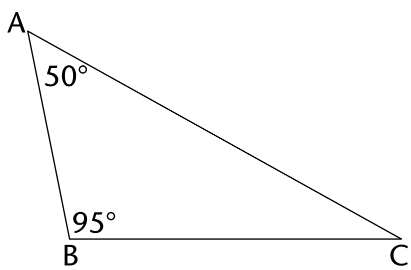
You can shorten the following reasons in the ways shown:

- Sum of interior angles (∠s) of a triangle (Δ) = 180°: **Interior ∠s of Δ**
- Isosceles triangle has 2 sides and 2 angles equal: **Isosceles Δ**
- Equilateral triangle has 3 sides and 3 angles equal: **Equilateral Δ**
- Angles forming a straight line = 180°: **Straight line**

WORKING OUT UNKNOWN ANGLES AND SIDES

Find the sizes of unknown angles and sides in the following triangles. Always give reasons for every statement.

1. What is the size of $\hat{C}$?



$\hat{A} + \hat{B} + \hat{C} =$ .... [Interior ∠s of a Δ]

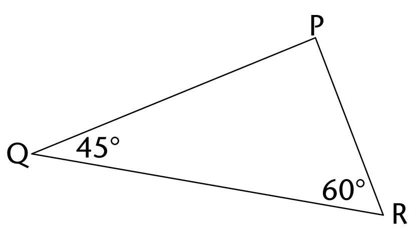
$50° +$ .... $+ \hat{C} =$ ....

$145° + \hat{C} =$ ....

$\hat{C} =$ .... $- 145°$

$\hat{C} =$ ....

2. Determine the size of $\hat{P}$.



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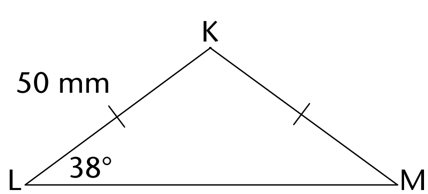
3. (a) What is the length of KM?
   (b) Find the size of $\hat{K}$.



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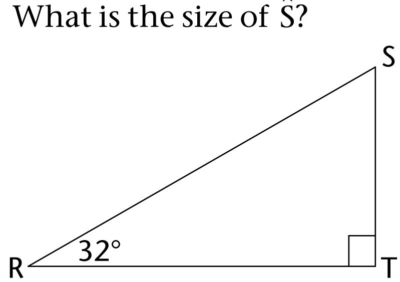
4. What is the size of $\hat{S}$?



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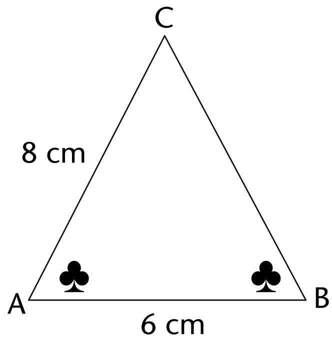
5. (a) Find CB.
   (b) Find $\hat{C}$ if $\hat{A} = 50°$.



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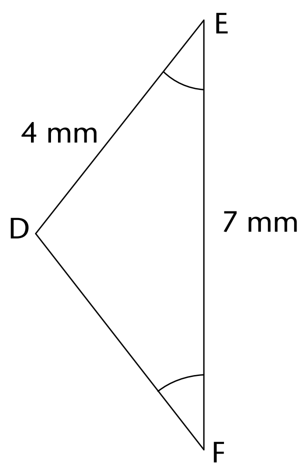
6. (a) Find DF.
   (b) Find $\hat{E}$ if $\hat{D} = 100°$.



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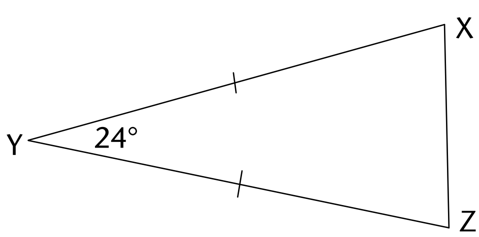
WORKING OUT MORE UNKNOWN ANGLES AND SIDES

1. Calculate the size of $\hat{X}$ and $\hat{Z}$.



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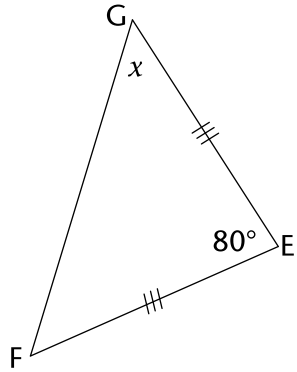
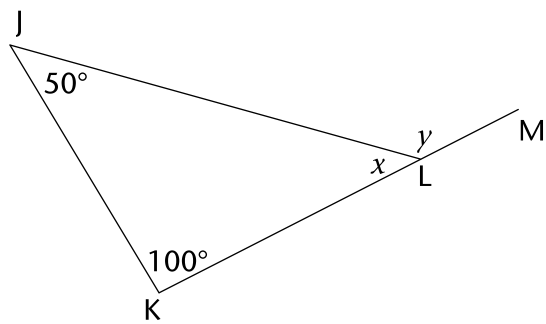
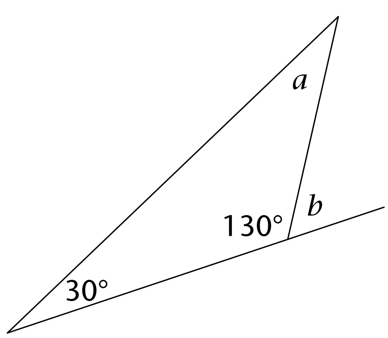
2. Calculate the size of x.



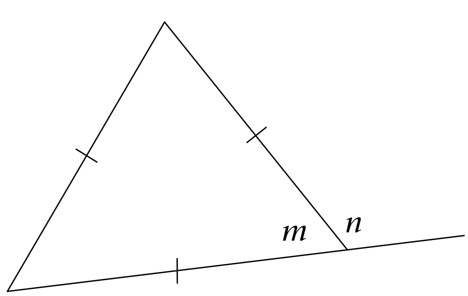
3. KLM is a straight line. Calculate the size of x and y.



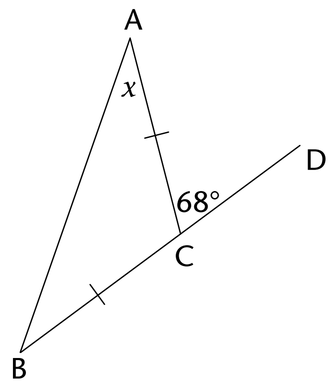
4. Angle b and an angle with size 130° form a straight angle. Calculate the size of a and b.



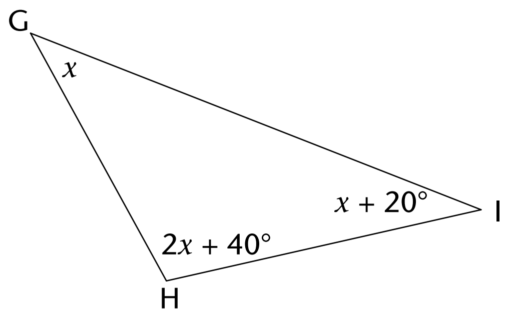
5. m and n form a straight angle. Calculate the size of m and n.



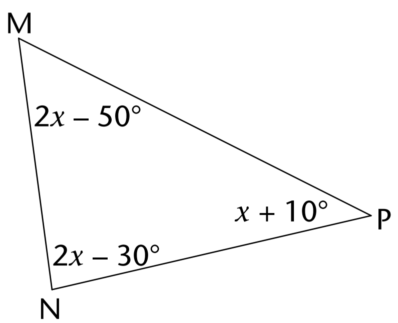
6. BCD is a straight line segment. Calculate the size of x.



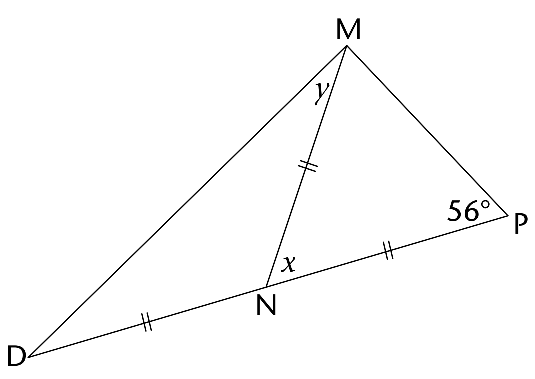
7. Calculate the size of x and then the size of $\hat{H}$.



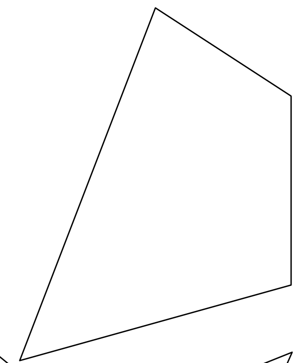
8. Calculate the size of $\hat{N}$.

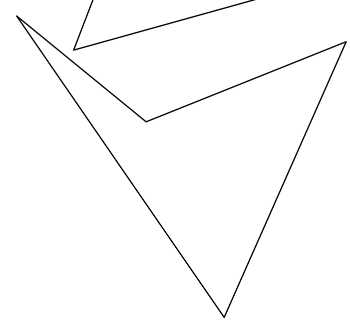


9. DNP is a straight line. Calculate the size of x and of y.



11.3 Types of quadrilaterals and their properties





A quadrilateral is a figure with four straight sides which meet at four vertices. In many quadrilaterals all the sides are of different lengths and all the angles are of different sizes.

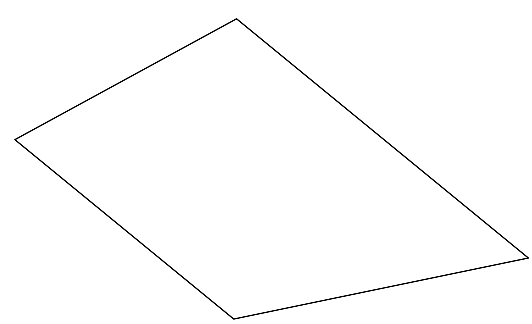
You have previously worked with these types of quadrilaterals, in which some sides have the same lengths, and some angles may be of the same size.

- parallelograms
- rectangles
- kites
- rhombuses
- squares
- trapeziums

THE PROPERTIES OF DIFFERENT TYPES OF QUADRILATERALS

1. In each question below, different examples of a certain type of quadrilateral are given. In each case identify which kind of quadrilateral it is. Describe the properties of each type by making statements about the lengths and directions of the sides and the sizes of the angles of each type. You may have to take some measurements to be able to do this.

Question 1(a)



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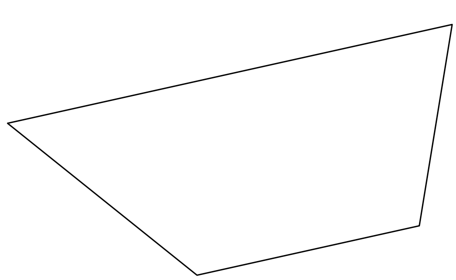
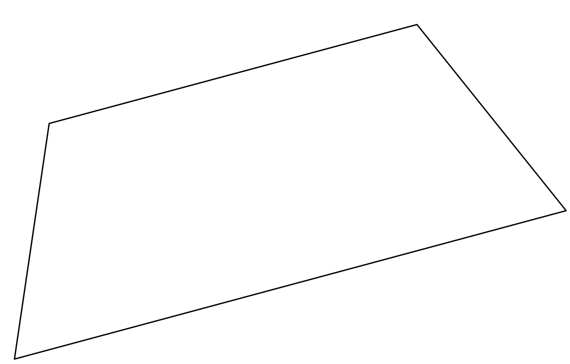
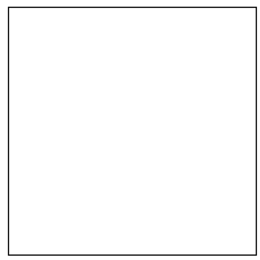
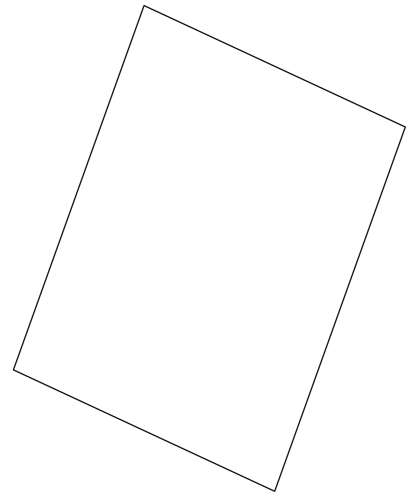


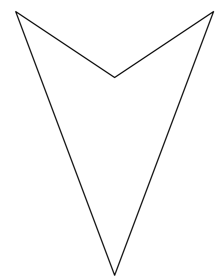


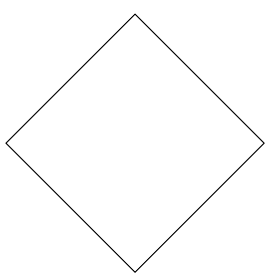
Question 1(b)

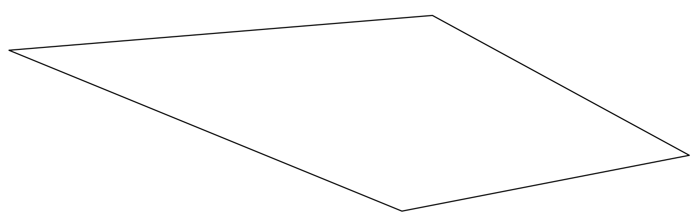




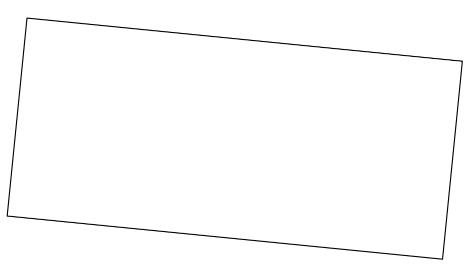
Question 1(c)

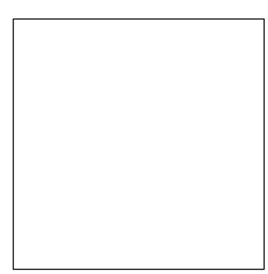




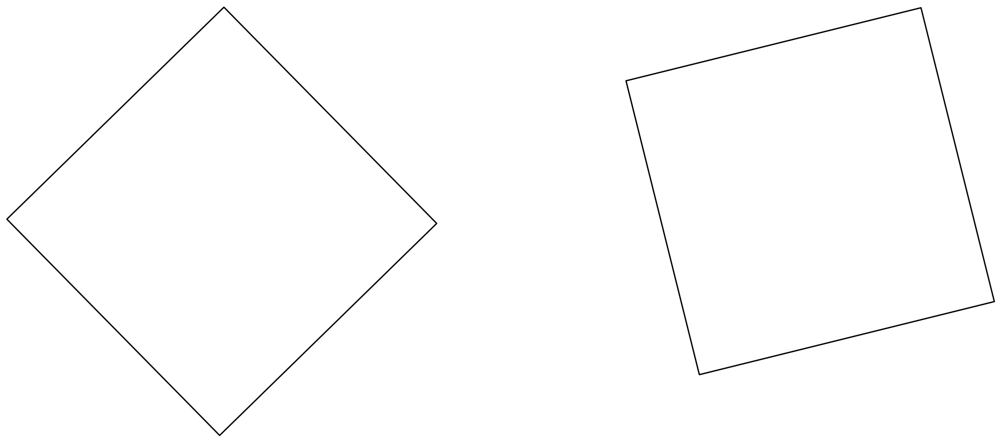


Question 1(d)

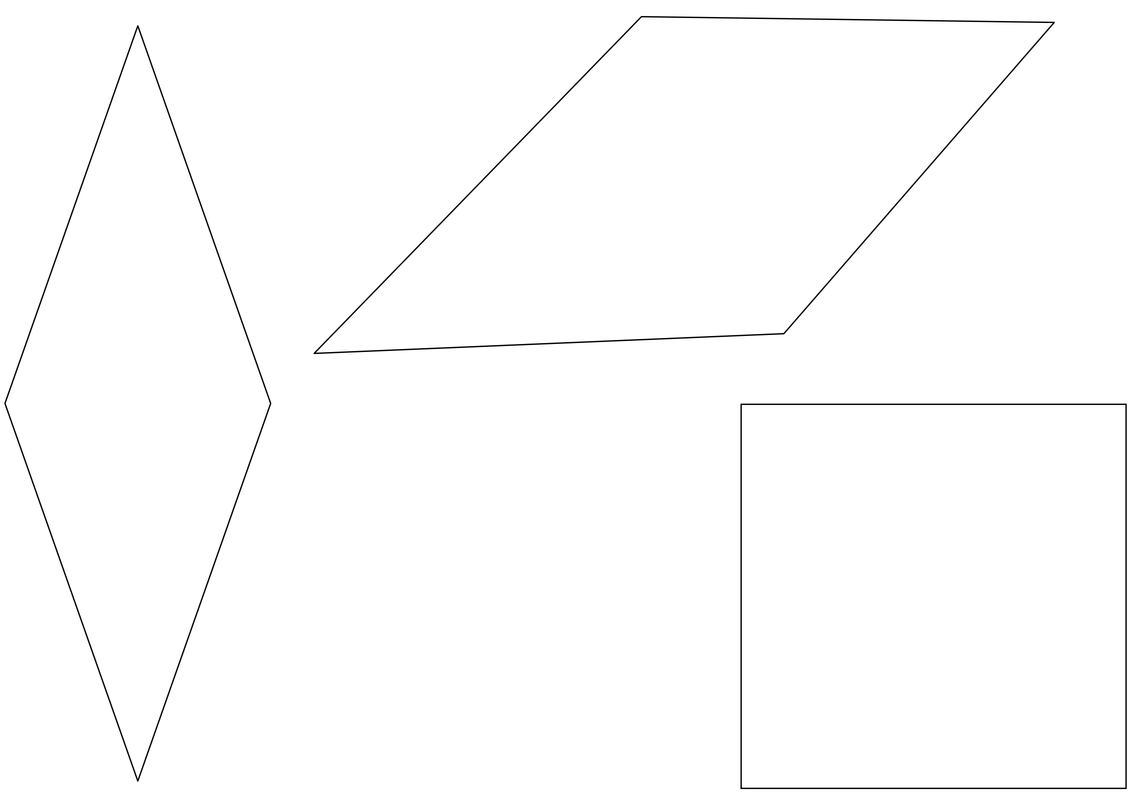




Question 1(e)



Question 1(f)



2. Use your completed lists and the drawings in question 1 to determine if the following statements are true (T) or false (F).

(a) A rectangle is a parallelogram. ....
(b) A square is a parallelogram. ....
(c) A rhombus is a parallelogram. ....
(d) A kite is a parallelogram. ....
(e) A trapezium is a parallelogram. ....
(f) A square is a rhombus. ....
(g) A square is a rectangle. ....
(h) A square is a kite. ....
(i) A rhombus is a kite. ....
(j) A rectangle is a rhombus. ....
(k) A rectangle is a square. ....

A **convention** is something (such as a definition or method) that most people agree on, accept and follow.

If a quadrilateral has *all* the properties of another quadrilateral, you can define it in terms of the other quadrilateral, as you have found above.

3. Here are some conventional definitions of quadrilaterals:
   - A **parallelogram** is a quadrilateral with two opposite sides parallel.
   - A **rectangle** is a parallelogram that has all four angles equal to 90°.
   - A **rhombus** is a parallelogram with all four sides equal.
   - A **square** is a rectangle with all four sides equal.
   - A **trapezium** is a quadrilateral with one pair of opposite sides parallel.
   - A **kite** is a quadrilateral with two pairs of adjacent sides equal.

   Write down other definitions that work for these quadrilaterals.

   (a) Rectangle: ....

   ....

   (b) Square: ....

   ....

   (c) Rhombus: ....

   ....

   (d) Kite: ....

   ....

   (e) Trapezium: ....

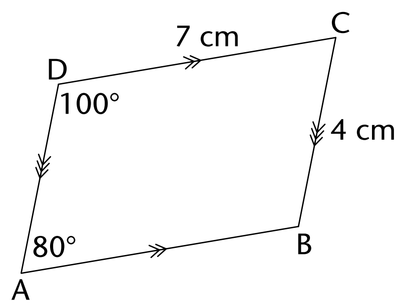
   ....

11.4 Unknown angles and sides of quadrilaterals

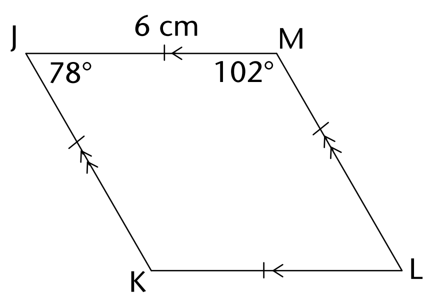
FINDING UNKNOWN ANGLES AND SIDES

Find the length of all the **unknown sides** and **angles** in the following quadrilaterals. Give reasons to justify your statements. (Also recall that the sum of the angles of a quadrilateral is 360°.)

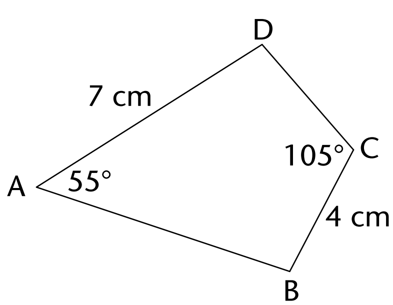
1.



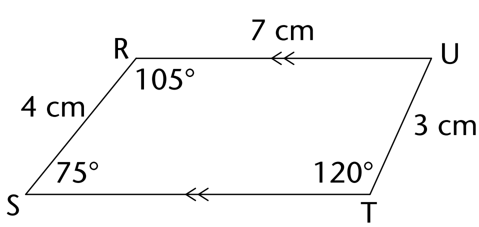
2.



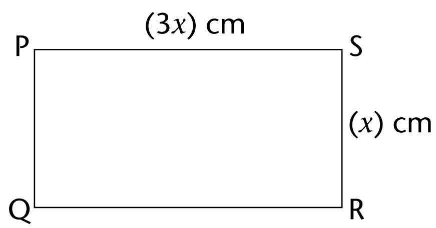
3. ABCD is a kite.



4. The perimeter of RSTU is 23 cm.



5. PQRS is a rectangle and has a perimeter of 40 cm.



11.5 Congruency

WHAT IS CONGRUENCY?

1. ΔABC is reflected in the vertical line (mirror) to give ΔKLM.
   Are the sizes and shapes of the two triangles exactly the same?

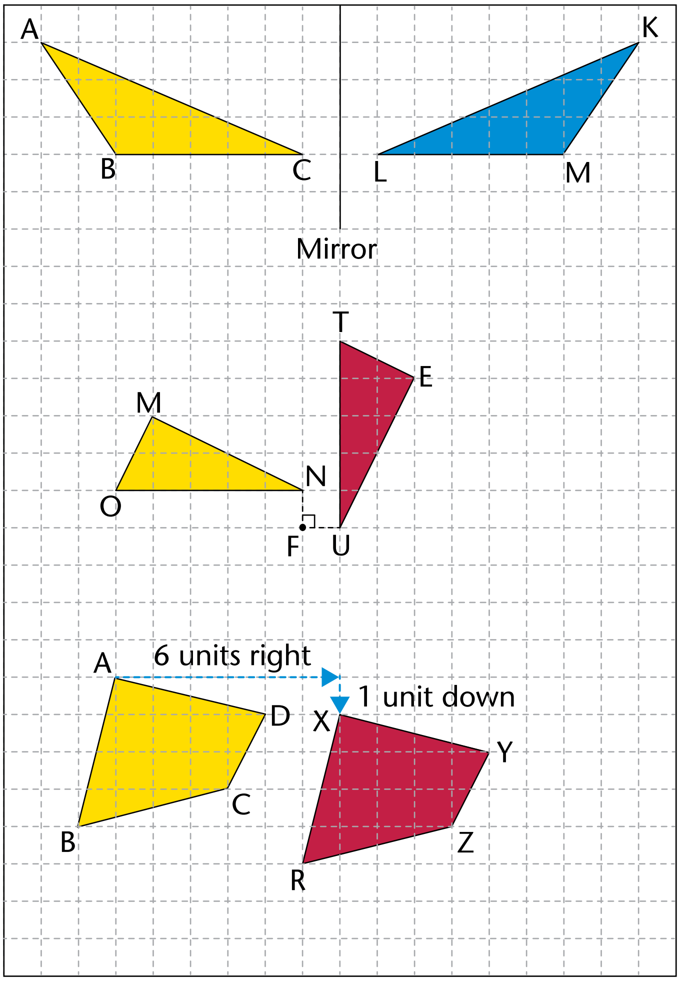
   ....

2. ΔMON is rotated 90° around point F to give you ΔTUE.
   Are the sizes and shapes of ΔMON and ΔTUE exactly the same?

   ....

3. Quadrilateral ABCD is translated 6 units to the right and 1 unit down to give quadrilateral XRZY.
   Are ABCD and XRZY exactly the same?

   ....



In the previous activity, each of the figures was transformed (reflected, rotated or translated) to produce a second figure. The second figure in each pair has **the same angles, side lengths, size and area** as the first figure. The second figure is thus an **accurate copy** of the first figure.

> When one figure is an image of another figure, we say that the two figures are **congruent**.
> The symbol for congruent is: ≡

The word **congruent** comes from the Latin word *congruere*, which means "to agree". Figures are congruent if they match up perfectly when laid on top of each other.

Notation of congruent figures

When we name shapes that are congruent, we name them so that the matching, or corresponding, angles are in the same order. For example, in **ΔABC** and **ΔKLM** on the previous page:

$\hat{A}$ is congruent to (matches and is equal to) $\hat{K}$.
$\hat{B}$ is congruent to $\hat{M}$.
$\hat{C}$ is congruent to $\hat{L}$.

We therefore use this notation: **ΔABC ≡ ΔKML**.

We cannot assume that, when the angles of polygons are equal, the polygons are congruent. You will learn about the conditions of congruence in Grade 9.

Similarly for the other pairs of figures on the previous page:
ΔMON ≡ ΔETU and **ABCD ≡ XRZY**.

The notation of congruent figures also shows which sides of the two figures correspond and are equal. For example, ΔABC ≡ ΔKML shows that:

AB = KM, BC = ML and AC = KL

The incorrect notation ΔABC ≡ ΔKLM will show the following incorrect information:

$\hat{B} = \hat{L}$, $\hat{C} = \hat{M}$, AB = KL, and AC = KM.

IDENTIFYING CONGRUENT ANGLES AND SIDES

Write down which angles and sides are equal between each pair of congruent figures.

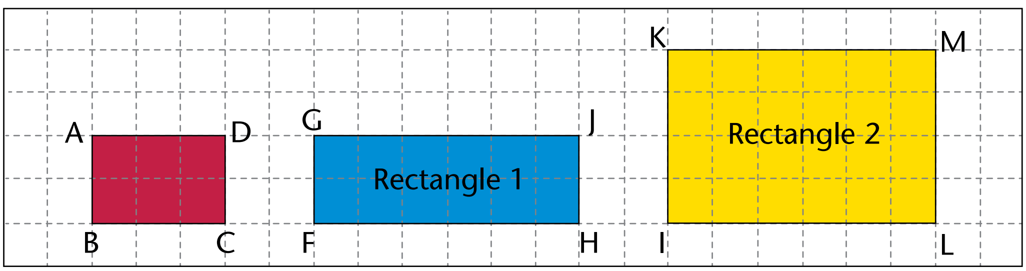
1. ΔPQR ≡ ΔUCT	2. ΔKLM ≡ ΔUWC
3. ΔGHI ≡ ΔQRT	4. ΔKJL ≡ ΔPOQ

11.6 Similarity

In Grade 7, you learnt that two figures are **similar** when they have the **same shape** (their angles are equal) but they may be **different sizes**. The sides of one figure are proportionally longer or shorter than the sides of the other figure; that is, the length of each side is multiplied or divided by the same number. We say that one figure is an enlargement or a reduction of the other figure.

CHECKING FOR SIMILARITY

1. Look at the rectangles below and answer the questions that follow.



(a) Look at rectangle 1 and ABCD:

How many times is FH longer than BC? ....

How many times is GF longer than AB? ....

(b) Look at rectangle 2 and ABCD:

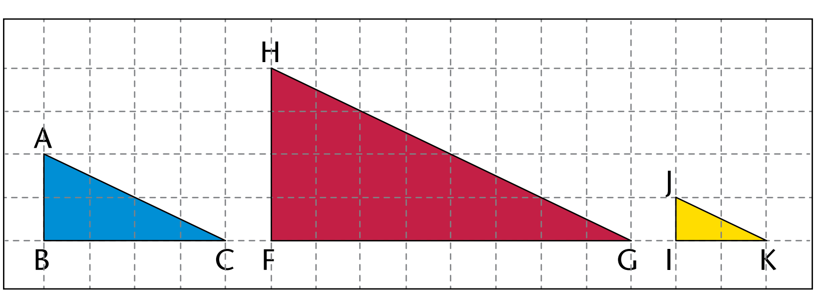
How many times is IL longer than BC? ....

How many times is LM longer than CD? ....

(c) Is rectangle 1 or rectangle 2 an enlargement of rectangle ABCD? Explain your answer.

....

2. Look at the triangles below and answer the questions that follow.



(a) How many times is:

- FG longer than BC? ....
- HF longer than AB? ....
- HG longer than AC? ....
- IK shorter than BC? ....
- JI shorter than AB? ....
- JK shorter than AC? ....

(b) Is ΔHFG an enlargement of ΔABC? Explain your answer.

....

(c) Is ΔJIK a reduction of ΔABC? Explain your answer.

....

In the previous activity, rectangle KILM is an enlargement of rectangle ABCD. Therefore, ABCD is similar to KILM. The symbol for 'is similar to' is: |||. So we write: ABCD ||| KILM.

The triangles on the previous page are also similar. ΔHFG is an enlargement of ΔABC and ΔJIK is a reduction of ΔABC.

In ΔABC and ΔHFG, $\hat{A} = \hat{H}$, $\hat{B} = \hat{F}$ and $\hat{C} = \hat{G}$. We therefore write it like this: ΔABC ||| ΔHFG.

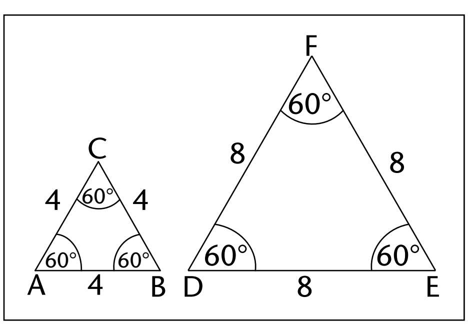
In the same way, ΔABC ||| ΔJIK.

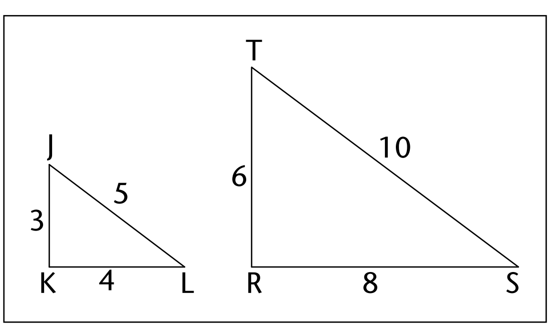
When you enlarge or reduce a polygon, you need to enlarge or reduce all its sides proportionally, or by the same ratio. This means that you multiply or divide each length by the same number.

Similar figures are figures that have the same angles (same shape) but are not necessarily the same size.

USING PROPERTIES OF SIMILAR AND CONGRUENT FIGURES

1. Are the triangles in each pair similar or congruent? Give a reason for each answer.

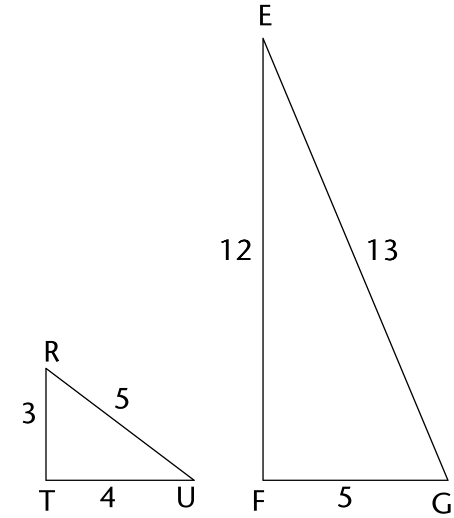




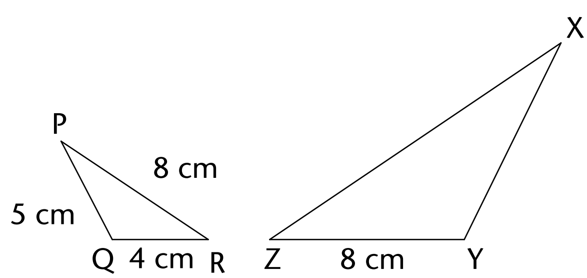
....

....

2. Is ΔRTU ||| ΔEFG? Give a reason for your answer.



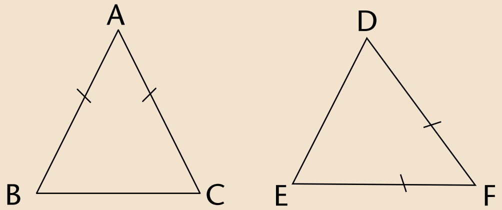
3. ΔPQR ||| ΔXYZ. Determine the length of XZ and XY.
(ZY is twice as long as QR.)



4. Are the following statements true or false? Explain your answers.
   (a) Figures that are congruent are similar.
   (b) Figures that are similar are congruent.
   (c) All rectangles are similar.
   (d) All squares are similar.

WORKSHEET

1. Study the triangles below and answer the following questions:



   (a) Tick the correct answer. ΔABC is:

   - ☐ acute and equilateral
   - ☐ obtuse and scalene
   - ☐ acute and isosceles
   - ☐ right-angled and isosceles.

   (b) If AB = 40 mm, what is the length of AC? ....

   (c) If $\hat{B}$ = 80°, what is the size of $\hat{C}$ and of $\hat{A}$? ....

   (d) ΔABC ≡ ΔFDE. Name all the sides in the two triangles that are equal to AB.

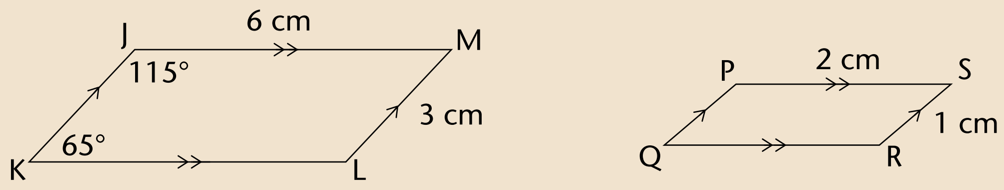
   ....

   (e) Name the side that is equal to DE. ....

   (f) If $\hat{F}$ is 40°, what is the size of $\hat{B}$? ....

   ....

2. Look at figures JKLM and PQRS. (Give reasons for your answers below.)



   (a) What type of quadrilateral is JKLM? ....

   (b) Is JKLM ||| PQRS? ....

   (c) What is the size of $\hat{L}$? ....

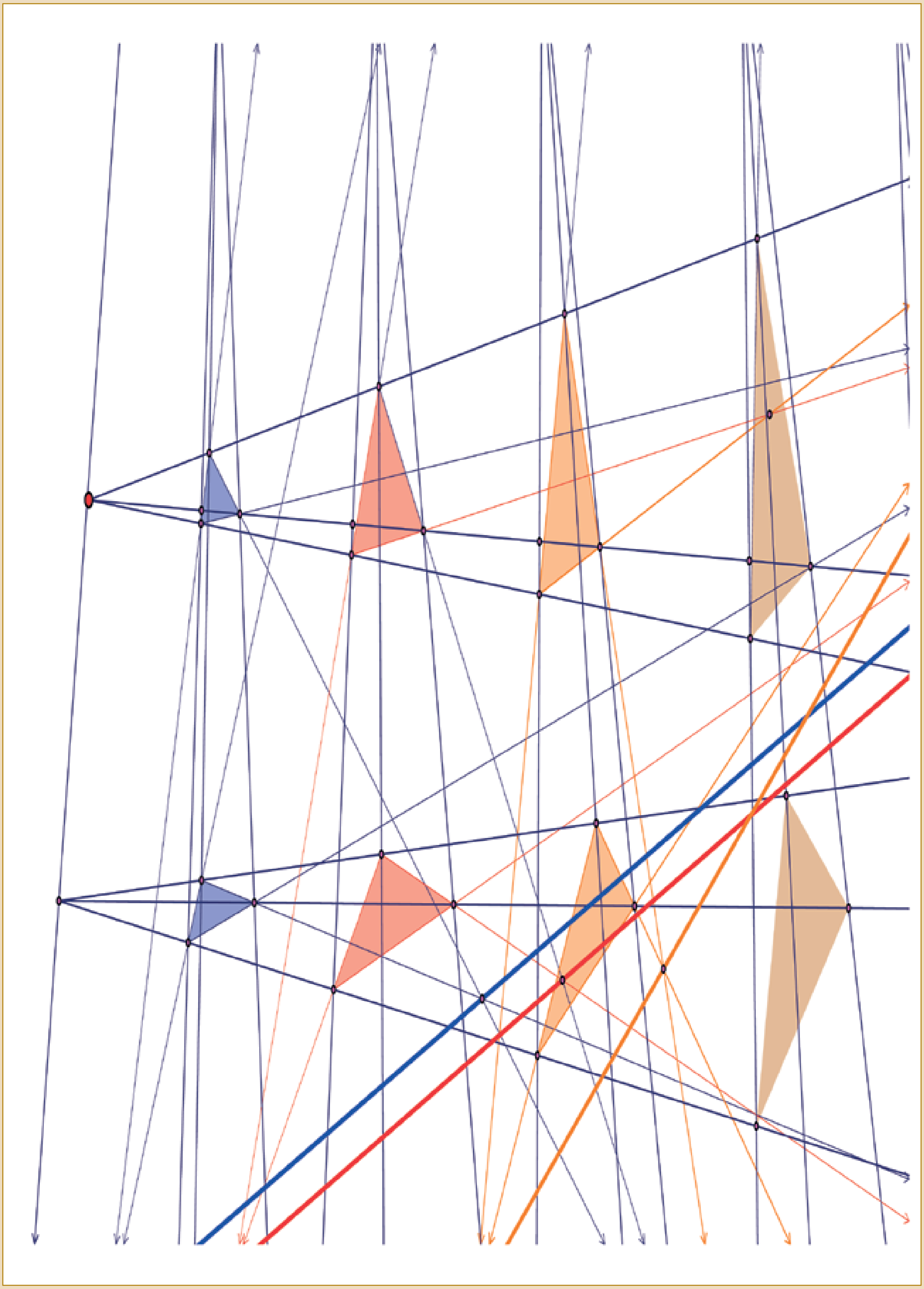
   (d) What is the size of $\hat{S}$? ....

   ....

   (e) What is the length of KL? ....

Chapter 12
Geometry of straight lines

In this chapter, you will explore the relationships between pairs of angles that are created when straight lines intersect (meet or cross). You will examine the pairs of angles that are formed by perpendicular lines, by any two intersecting lines, and by a third line that cuts two parallel lines. You will come to understand what is meant by vertically opposite angles, corresponding angles, alternate angles and co-interior angles. You will be able to identify different angle pairs, and then use your knowledge to help you work out unknown angles in geometric figures.



12 Geometry of straight lines

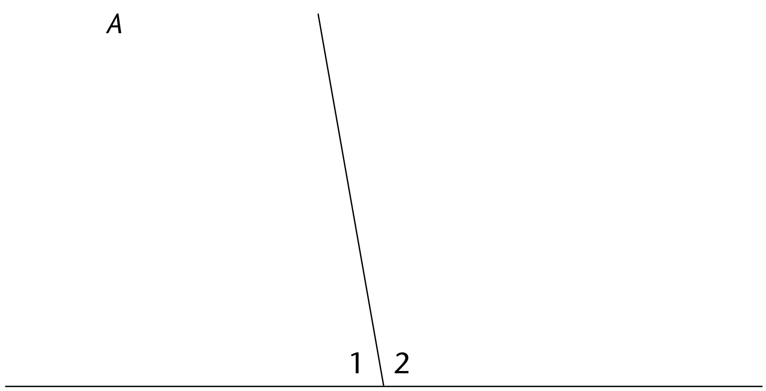
12.1 Angles on a straight line

SUM OF ANGLES ON A STRAIGHT LINE

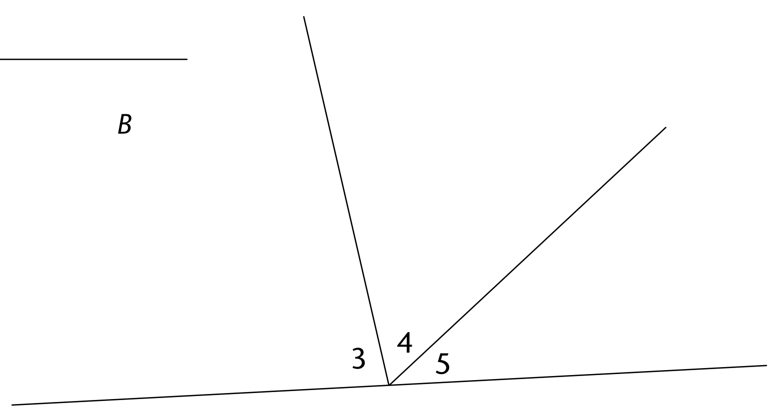
In the figures below, each angle is given a label from 1 to 5.

1. Use a protractor to measure the sizes of all the angles in each figure. Write your answers on each figure.

A



B



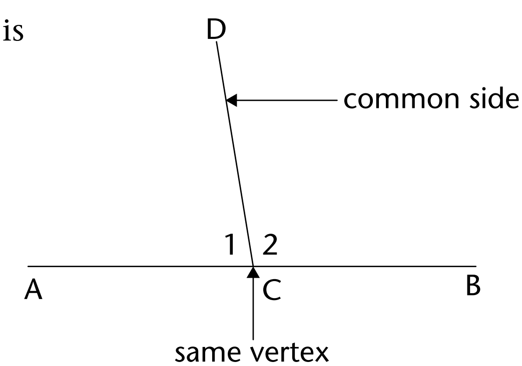
2. Use your answers to fill in the angle sizes below.

(a) $\hat{1} + \hat{2} = \ldots\ldots^\circ$ (b) $\hat{3} + \hat{4} + \hat{5} = \ldots\ldots^\circ$

The sum of angles that are formed on a straight line is equal to 180°. (We can shorten this property as: ∠s on a straight line.)

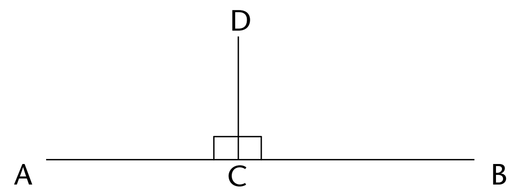
Two angles whose sizes add up to 180° are also called **supplementary** angles, for example $\hat{1} + \hat{2}$.

Angles that share a vertex and a common side are said to be **adjacent**. So $\hat{1} + \hat{2}$ are therefore also called **supplementary adjacent angles**.



When two lines are perpendicular, their adjacent supplementary angles are each equal to 90°.

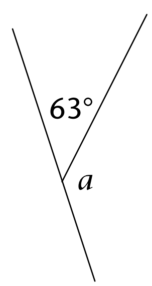
In the drawing alongside, $D\hat{C}A$ and $D\hat{C}B$ are adjacent supplementary angles because they are next to each other (adjacent) and they add up to 180° (supplementary).



FINDING UNKNOWN ANGLES ON STRAIGHT LINES

Work out the sizes of the unknown angles below. Build an equation each time as you solve these geometric problems. Always give a reason for every statement you make.

1. Calculate the size of a.

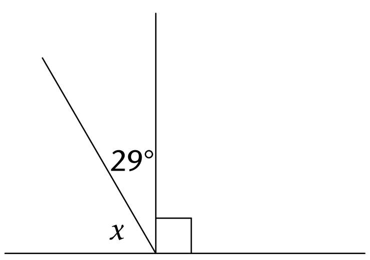


$a + 63° = ...$ [∠s on a straight line]

$a = ... - 63°$

$= .....$

2. Calculate the size of x.



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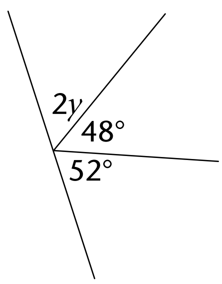
....

....

....

3. Calculate the size of y.



....

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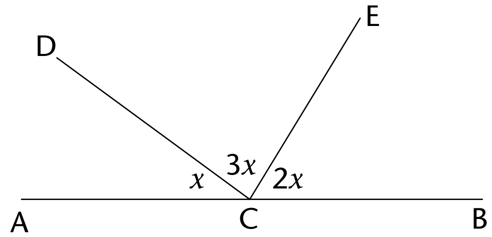
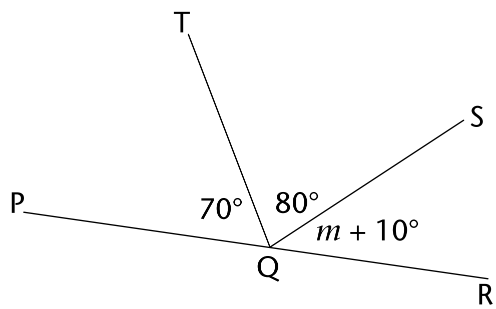
FINDING MORE UNKNOWN ANGLES ON STRAIGHT LINES

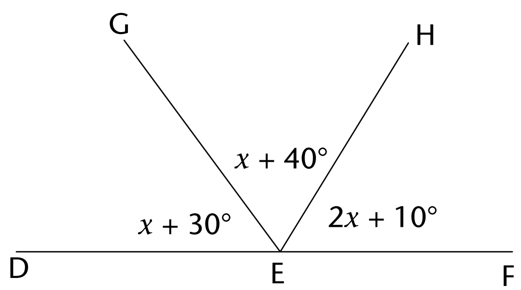
1. Calculate the size of:
   (a) x
   (b) $E\hat{C}B$



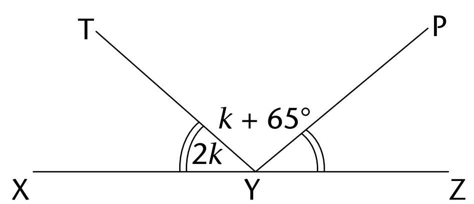
2. Calculate the size of:
   (a) m
   (b) $S\hat{Q}R$



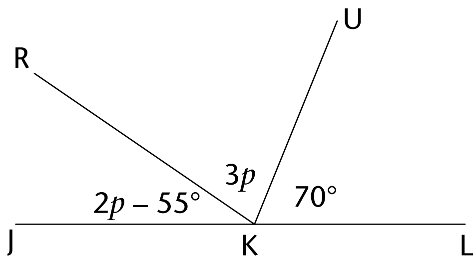
3. Calculate the size of:
   (a) x
   (b) $H\hat{E}F$



4. Calculate the size of:
   (a) k
   (b) $T\hat{Y}P$



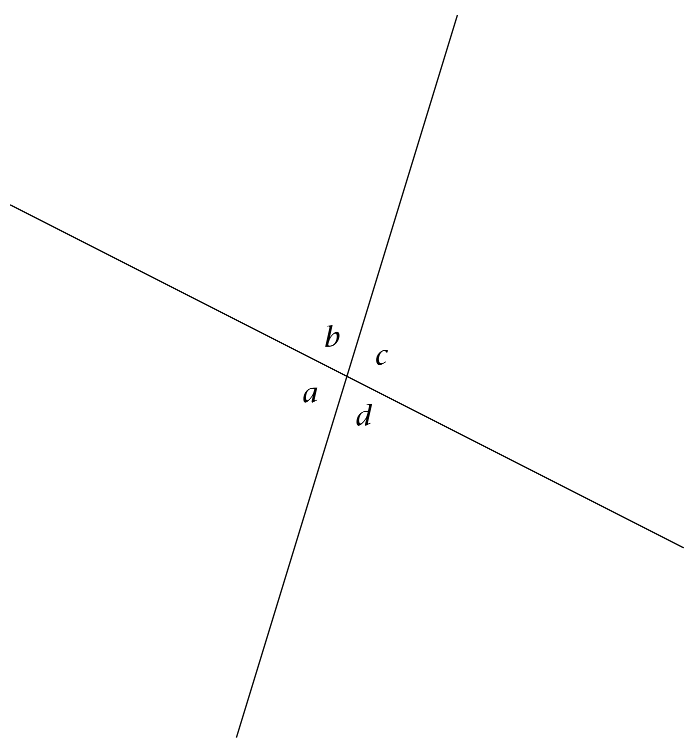
5. Calculate the size of:
   (a) p
   (b) $\text{J}\hat{\text{K}}\text{R}$



12.2 Vertically opposite angles

WHAT ARE VERTICALLY OPPOSITE ANGLES?

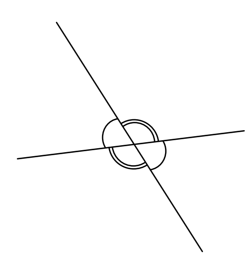
1. Use a protractor to measure the sizes of all the angles in the figure. Write your answers on the figure.



2. Notice which angles are equal and how these equal angles are formed.

Vertically opposite angles (**vert. opp. ∠s**) are the angles opposite each other when two lines intersect.

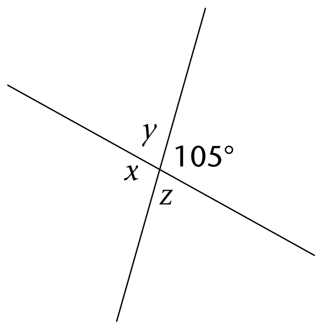
Vertically opposite angles are **always equal**.



FINDING UNKNOWN ANGLES

Calculate the sizes of the unknown angles in the following figures. Always give a reason for every statement you make.

1. Calculate x, y and z.



x = ...° [vert. opp. ∠s]

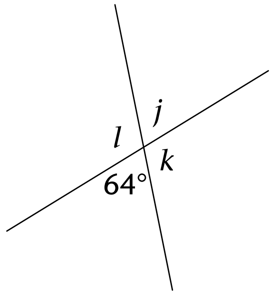
$y + 105°$ = ...° [∠s on a straight line]

y = ... − 105°

= .....

z = ... [vert. opp. ∠s]

2. Calculate j, k and l.



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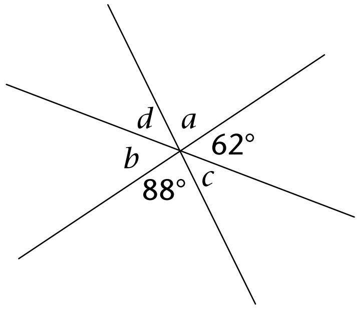
3. Calculate a, b, c and d.



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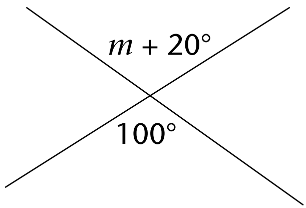
EQUATIONS USING VERTICALLY OPPOSITE ANGLES

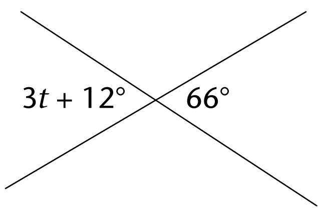
Vertically opposite angles are always equal. We can use this property to build an equation. Then we solve the equation to find the value of the unknown variable.

1. Calculate the value of *m*.



$$\begin{aligned} m + 20° &= 100° && \text{[vert. opp. } \angle\text{s]} \\ m &= 100° - 20° \\ &= \ldots\ldots \end{aligned}$$

2. Calculate the value of *t*.



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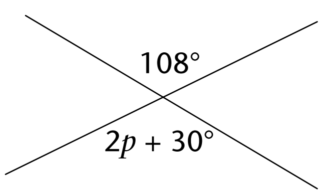
3. Calculate the value of *p*.



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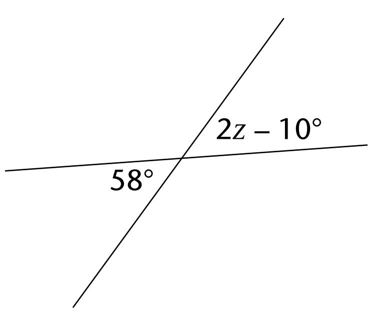
4. Calculate the value of *z*.



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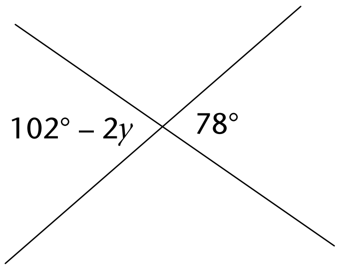
5. Calculate the value of y.



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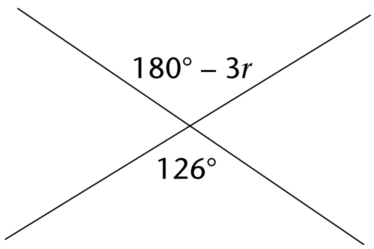
6. Calculate the value of r.



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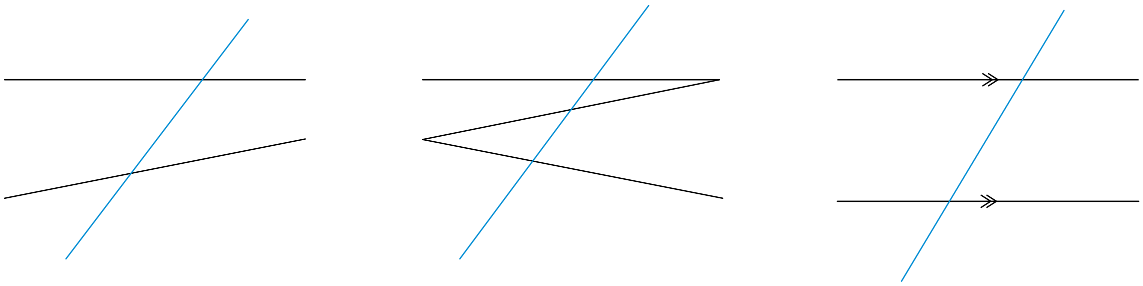
12.3 Lines intersected by a transversal

PAIRS OF ANGLES FORMED BY A TRANSVERSAL

A **transversal** is a line that crosses at least two other lines.

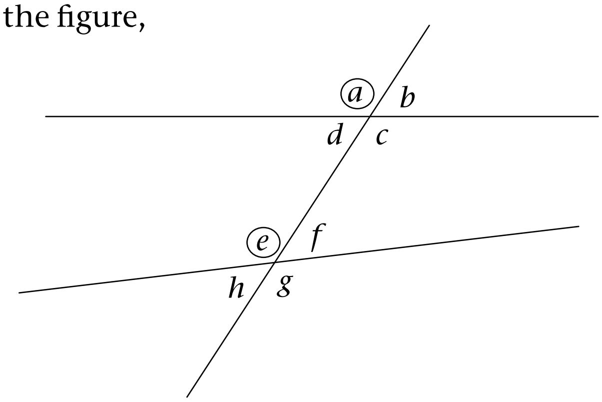
The blue line is the transversal



When a transversal intersects two lines, we can compare the sets of angles on the two lines by looking at their positions.

The angles that lie on the same side of the transversal and are in matching positions are called **corresponding angles** (**corr. ∠s**). In the figure, these are corresponding angles:

- *a* and *e*
- *b* and *f*
- *d* and *h*
- *c* and *g*.



1. In the figure, *a* and *e* are both left of the transversal and above a line.

   Write down the location of the following corresponding angles. The first one is done for you.

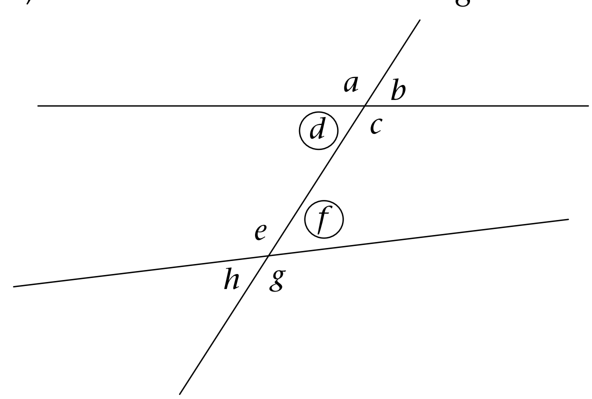
   b and *f*: Right of the transversal and above the lines. ....

   d and *h*: ....

   c and *g*: ....

Alternate angles (**alt. ∠s**) lie on opposite sides of the transversal, but are not adjacent or vertically opposite. When the alternate angles lie between the two lines, they are called **alternate interior angles**. In the figure, these are alternate interior angles:

- *d* and *f*
- *c* and *e*.



When the alternate angles lie outside of the two lines, they are called **alternate exterior angles**. In the figure, these are alternate exterior angles:

- *a* and *g*
- *b* and *h*.

2. Write down the location of the following alternate angles:

   d and *f*: ....

   ....

   c and *e*: ....

   ....

   a and *g*: ....

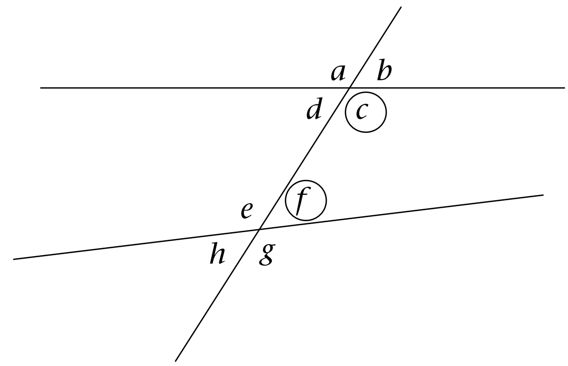
   ....

   b and *h*: ....

   ....

Co-interior angles (co-int. ∠s) lie on the same side of the transversal and between the two lines. In the figure, these are co-interior angles:

- *c* and *f*
- *d* and *e*.



3. Write down the location of the following co-interior angles:

   d and *e*: ....

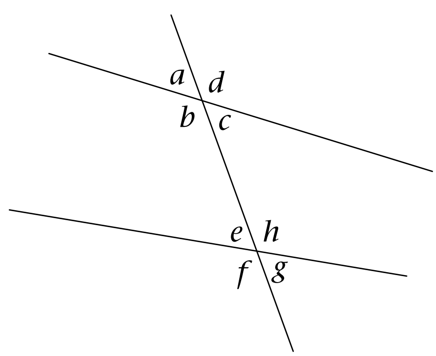
   ....

   c and *f*: ....

   ....

IDENTIFYING TYPES OF ANGLES

Two lines are intersected by a transversal as shown below.



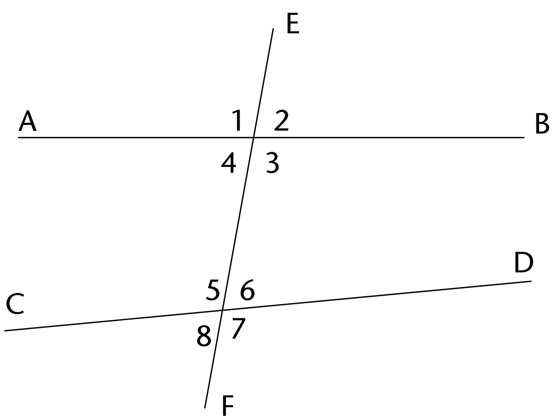
Write down the following pairs of angles:

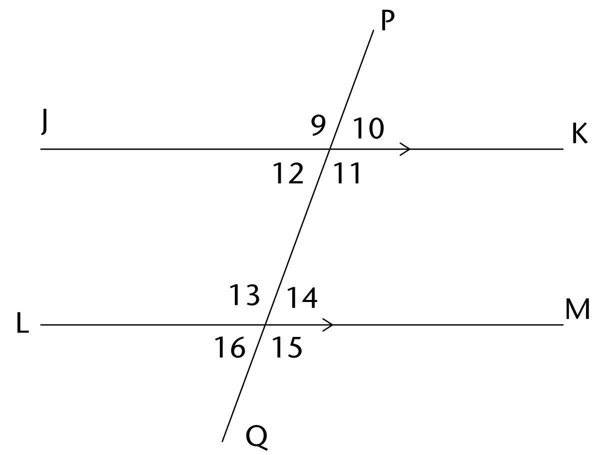
1. two pairs of corresponding angles: ....
2. two pairs of alternate interior angles: ....
3. two pairs of alternate exterior angles: ....
4. two pairs of co-interior angles: ....
5. two pairs of vertically opposite angles: ....

12.4 Parallel lines intersected by a transversal

INVESTIGATING ANGLE SIZES

In the figure below left, EF is a transversal to AB and CD. In the figure below right, PQ is a transversal to parallel lines JK and LM.





1. Use a protractor to measure the sizes of all the angles in each figure. Write the measurements on the figures.
2. Use your measurements to complete the following table.

Angles	When two lines are not parallel	When two lines are parallel
Corr. ∠s	$\hat{1}$ = ...; $\hat{5}$ = ... $\hat{4}$ = ...; $\hat{8}$ = ... $\hat{2}$ = ...; $\hat{6}$ = ... $\hat{3}$ = ...; $\hat{7}$ = ...	$\hat{9}$ = ...; $\widehat{13}$ = ... $\widehat{12}$ = ...; $\widehat{16}$ = ... $\widehat{10}$ = ...; $\widehat{14}$ = ... $\widehat{11}$ = ...; $\widehat{15}$ = ...
Alt. int. ∠s	$\hat{4}$ = ...; $\hat{6}$ = ... $\hat{3}$ = ...; $\hat{5}$ = ...	$\widehat{12}$ = ...; $\widehat{14}$ = ... $\widehat{11}$ = ...; $\widehat{13}$ = ...
Alt. ext. ∠s	$\hat{1}$ = ...; $\hat{7}$ = ... $\hat{2}$ = ...; $\hat{8}$ = ...	$\hat{9}$ = ...; $\widehat{15}$ = ... $\widehat{10}$ = ...; $\widehat{16}$ = ...
Co-int. ∠s	$\hat{4} + \hat{5}$ = ... $\hat{3} + \hat{6}$ = ...	$\widehat{12} + \widehat{13}$ = ... $\widehat{11} + \widehat{14}$ = ...

3. Look at your completed table in question 2. What do you notice about the angles formed when a transversal intersects parallel lines?

....

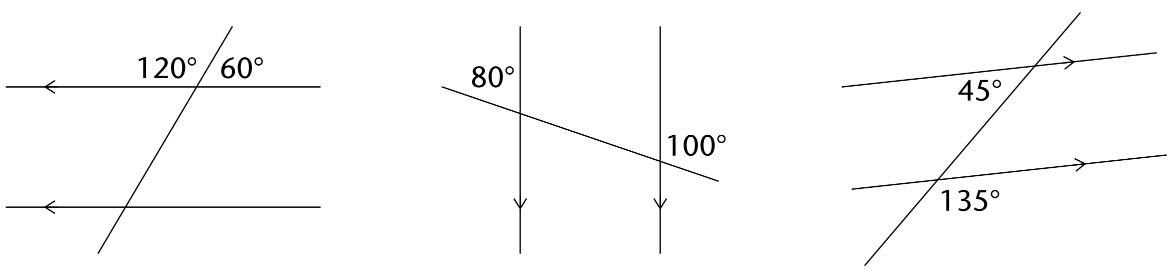
....

When lines are parallel:

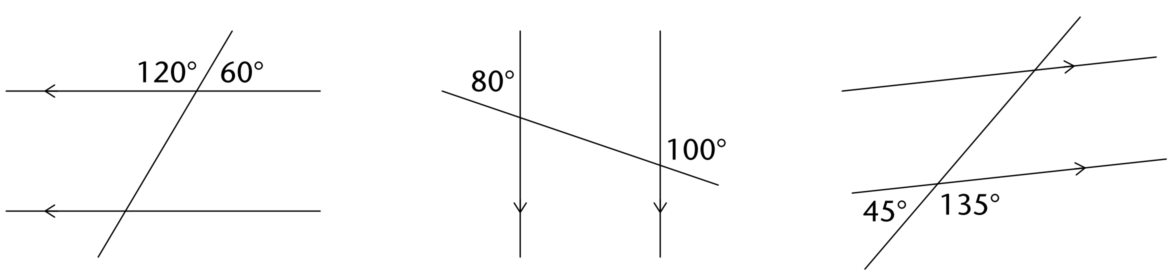
- corresponding angles are equal
- alternate interior angles are equal
- alternate exterior angles are equal
- co-interior angles add up to 180°.

IDENTIFYING ANGLES ON PARALLEL LINES

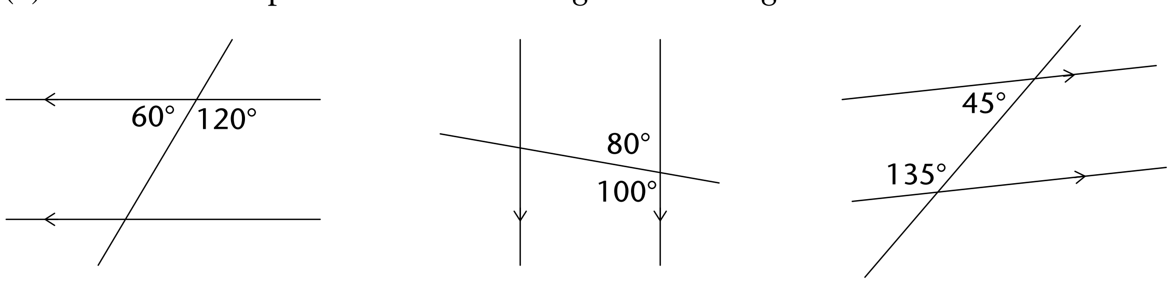
1. Fill in the corresponding angles to those given.



2. Fill in the alternate exterior angles.

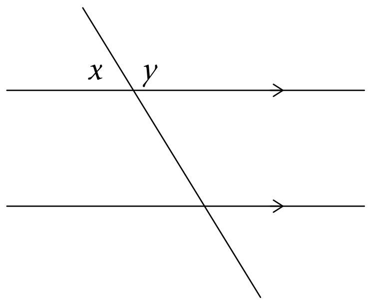


3. (a) Fill in the alternate interior angles.
   (b) Circle the two pairs of co-interior angles in each figure.

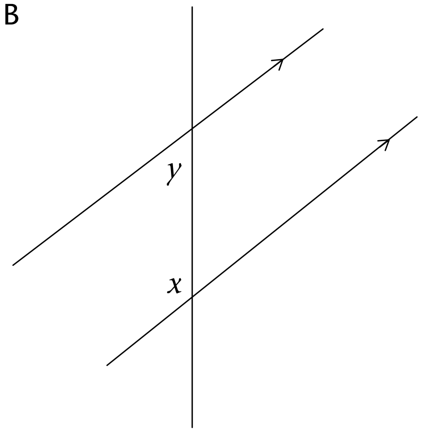


4. (a) Without measuring, fill in all the angles in the following figures that are equal to x and y.
   (b) Explain your reasons for each x and y that you filled in to your partner.

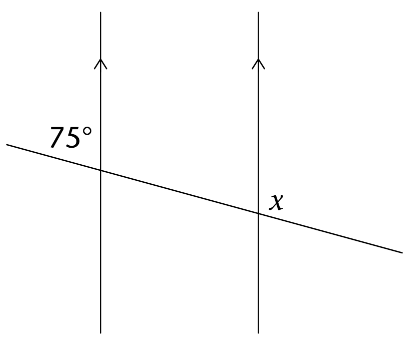
A

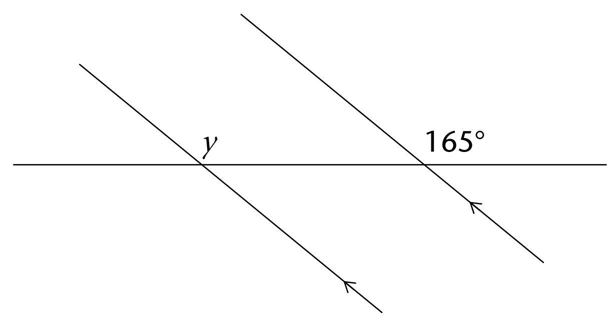


B



5. Give the value of x and y below.



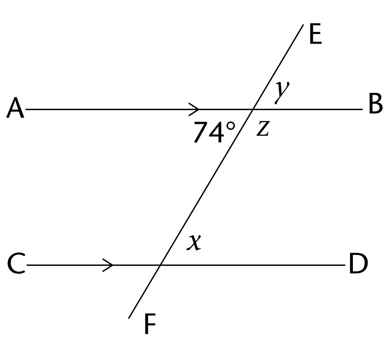


12.5 Finding unknown angles on parallel lines

WORKING OUT UNKNOWN ANGLES

Work out the sizes of the unknown angles. Give reasons for your answers. (The first one has been done as an example.)

1. Find the sizes of x, y and z.

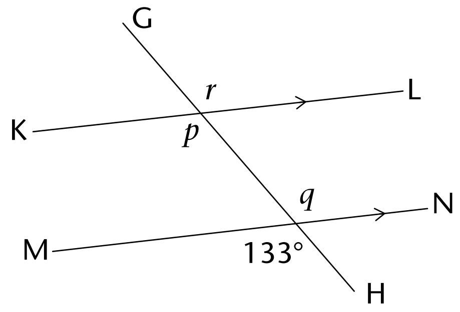


$x = 74°$ [alt. ∠ with given 74°; AB || CD]

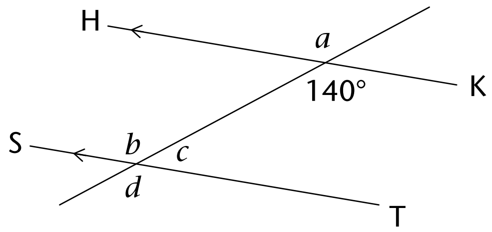
$y = 74°$ [corr. ∠ with x; AB || CD]
or $y = 74°$ [vert. opp. ∠ with given 74°]

$z = 106°$ [co-int. ∠ with x; AB || CD]
or $z = 106°$ [∠s on a straight line]

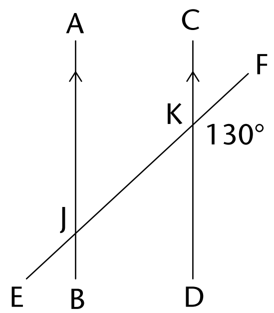
2. Work out the sizes of p, q and r.



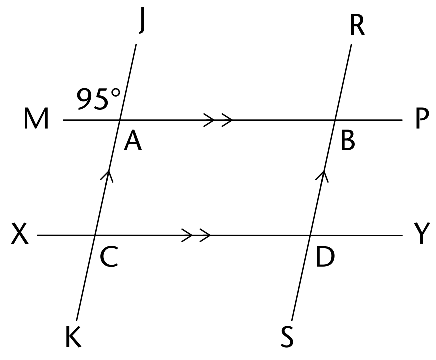
3. Find the sizes of a, b, c and d.



4. Find the sizes of all the angles in this figure.

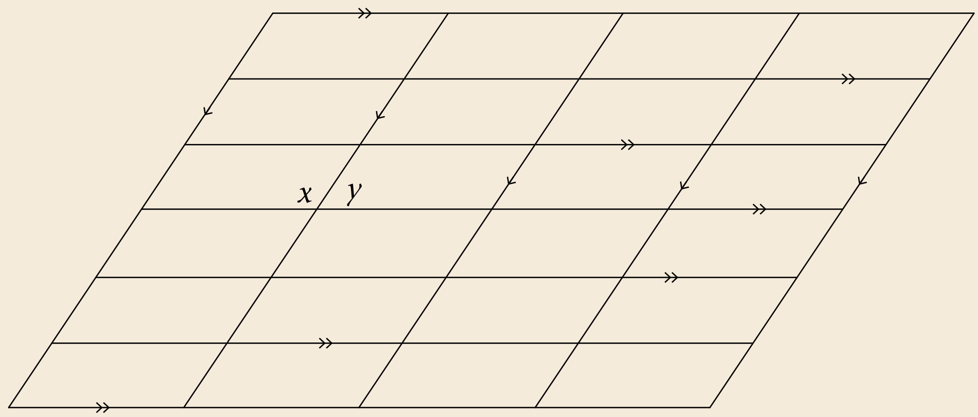


5. Find the sizes of all the angles. (Can you see two transversals and two sets of parallel lines?)



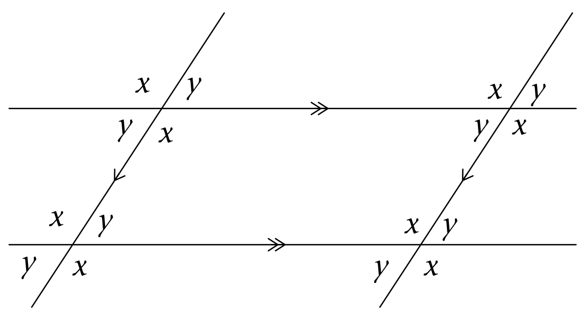
EXTENSION

Two angles in the following diagram are given as x and y. Fill in all the angles that are equal to x and y.



SUM OF THE ANGLES IN A QUADRILATERAL

The diagram below is a section of the previous diagram.



1. What kind of quadrilateral is in the diagram? Give a reason for your answer.

   ....

2. Look at the top left intersection. Complete the following equation:

   Angles around a point = 360°

   $\therefore \quad x + y + \ldots\ldots + \ldots\ldots = 360°$

3. Look at the interior angles of the quadrilateral. Complete the following equations:

   Sum of angles in the quadrilateral = $x + y + \ldots\ldots + \ldots\ldots$

   From question 2: $x + y + \ldots\ldots + \ldots\ldots = 360°$

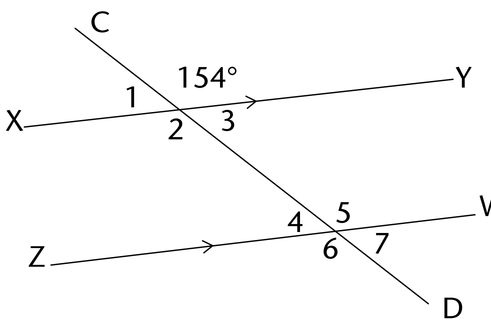
   $\therefore \quad$ Sum of angles in a quadrilateral = ...°

Can you think of another way to use the diagram above to work out the sum of the angles in a quadrilateral?

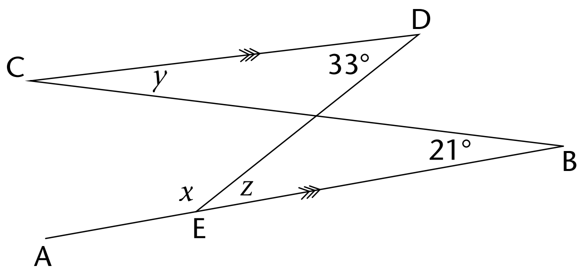
12.6 Solving more geometric problems

ANGLE RELATIONSHIPS ON PARALLEL LINES

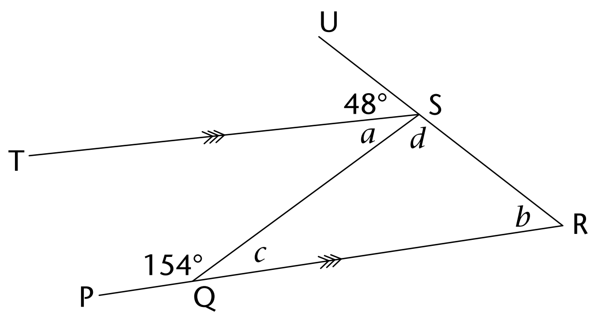
1. Calculate the sizes of $\hat{1}$ to $\hat{7}$.



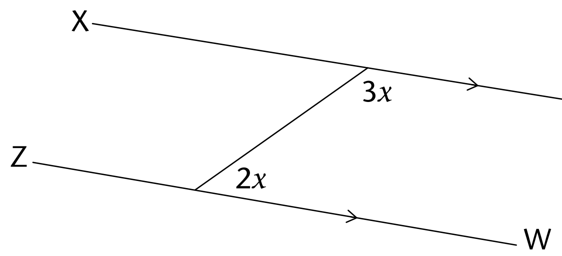
2. Calculate the sizes of x, y and z.



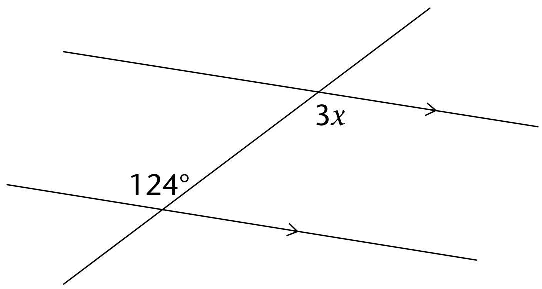
3. Calculate the sizes of a, b, c and d.



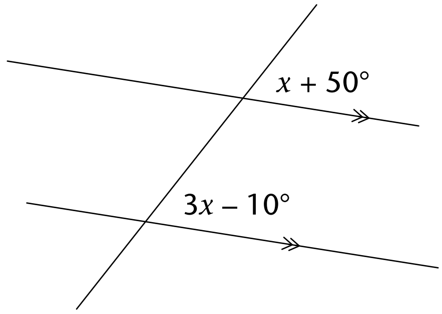
4. Calculate the size of x.



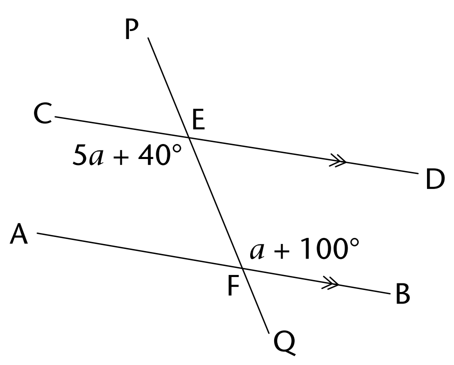
5. Calculate the size of x.



6. Calculate the size of x.

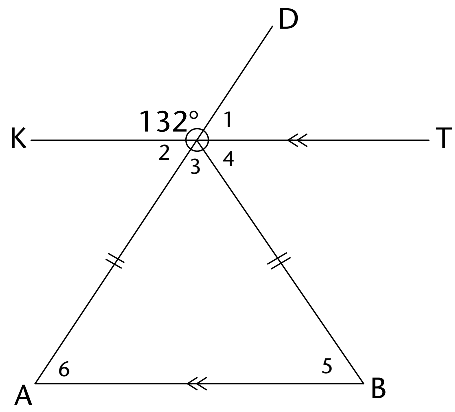


7. Calculate the sizes of a and $C\hat{E}P$.

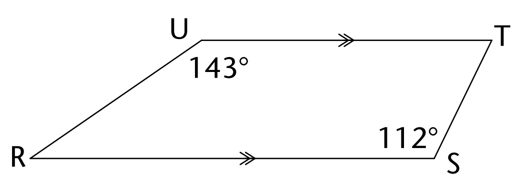


INCLUDING PROPERTIES OF TRIANGLES AND QUADRILATERALS

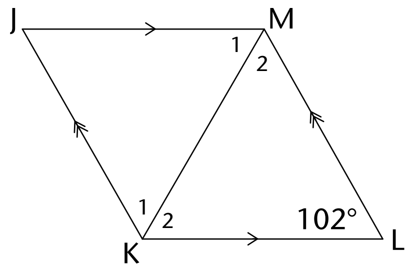
1. Calculate the sizes of $\hat{1}$ to $\hat{6}$.



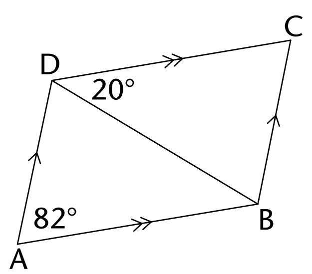
2. RSTU is a trapezium. Calculate the sizes of $\hat{T}$ and $\hat{R}$.



3. JKLM is a rhombus. Calculate the sizes of $J\widehat{M}L$, $\widehat{M}_2$ and $\hat{K}_1$.

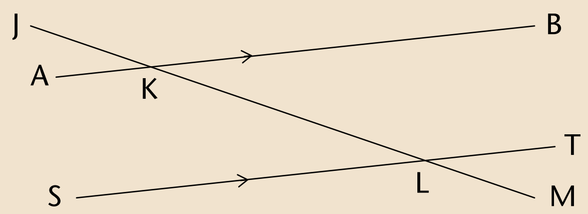


4. ABCD is a parallelogram. Calculate the sizes of $A\hat{D}B$, $A\hat{B}D$, $\hat{C}$ and $D\hat{B}C$.



WORKSHEET

1. Look at the drawing below. Name the items listed alongside.



(a) a pair of vertically opposite angles

....

(b) a pair of corresponding angles

....

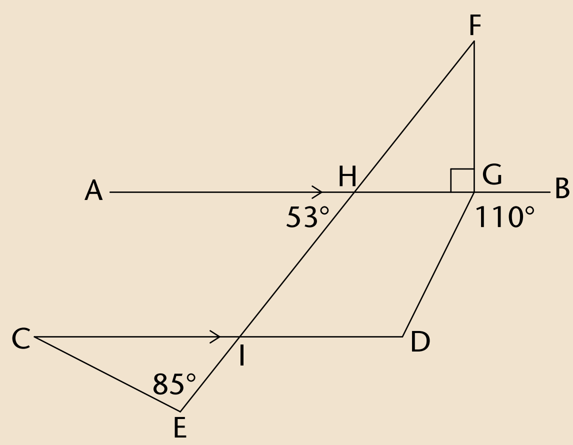
(c) a pair of alternate interior angles

....

(d) a pair of co-interior angles

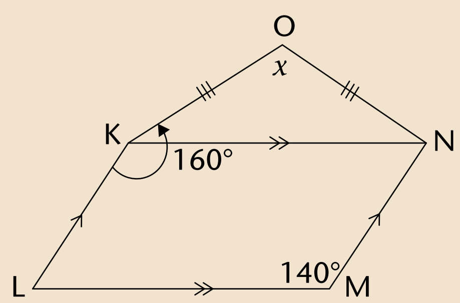
....

2. In the diagram, AB || CD. Calculate the sizes of $F\hat{H}G$, $\hat{F}$, $\hat{C}$ and $\hat{D}$. Give reasons for your answers.



3. In the diagram, OK = ON, KN || LM, KL || MN and $L\hat{K}O$ = 160°.

Calculate the value of x. Give reasons for your answers.



Term 2
Revision and assessment

Revision

Show all your steps in your working.

ALGEBRAIC EXPRESSIONS 2

1. Simplify:
   (a) $x^2 + x^2$

   ....

   (b) $m + m \times m + m$

   ....

   ....

   (c) $5ab - 7a^2 - 2a^2 + 11ba$

   ....

   ....

   (d) $(3ac^2)(-4a^2b)$

   ....

   (e) $(-4a^2b^3)^3$

   ....

   (f) $\left(\dfrac{-6x^2yz^4}{3xyz}\right)^2$

   ....

   ....

   (g) $\sqrt{\dfrac{100x^4}{81y^{64}}}$

   ....

   (h) $\sqrt{16c^2 + 9c^2}$

   ....

   ....

   ....

(i) $(2x + 3x)^3$

....

....

(j) $3x^2(4x^3 - 5)$

....

....

(k) $(4a - 7a)(a^2 - 2a - 5)$

....

....

(l) $\frac{9c^2de^3}{3c^2d^2e^2f}$

....

(m) $\frac{6b^2 - 3b^2}{2b^2}$

....

....

(n) $\frac{10x^2 - 5x + 1}{5}$

....

(o) $\frac{14x - 21x^2}{7x^2}$

....

....

....

2. Simplify the following expressions:

(a) $3(a + 2b) - 4(b - 2a)$

....

....

(b) $3 - 2(5x^2 + 6x - 2)$

....

....

(c) $2x(x^2 - x + 1) - 3(4 - x)$

....

....

....

(d) $(2a + b - 4c) - (5a + b - c)$

....

....

....

(e) $a\{2a^2[4 + 2(3a + 1)] - a\}$

....

....

....

3. If $a = 0$, $b = -2$, and $c = 3$, determine the value of the following without using a calculator. Show all working:

   (a) b^2c

   ....

   ....

   (b) $2b - b(ab - 5bc)$

   ....

   ....

   ....

   (c) $\frac{2b - c + 10a}{3c^2}$

   ....

   ....

   ....

4. If $y = -2$, find the value of $2y^3 - 4y + 3$

   ....

   ....

ALGEBRAIC EQUATIONS 2

1. Solve the following equations:
   (a) $-x = -7$

   ....

   ....

   (b) $2x = 24$

   ....

   ....

   (c) $3x - 6 = 0$

   ....

   ....

   (d) $2x + 5 = 3$

   ....

   ....

   (e) $3(x - 4) = -3$

   ....

   ....

   ....

   (f) $4(2x - 1) = 5(x - 2)$

   ....

   ....

   ....

2. Sello is x years old. Thlapo is 4 years older than Sello. The sum of their ages is 32.
   (a) Write this information in an equation using x as the variable.

   ....

   ....

   (b) Solve the equation to find Thlapo's age.

   ....

   ....

   ....

3. The length of a rectangle is $(2x + 8)$ cm and the width is 2 cm. The area of the rectangle is 12 cm^2.
   (a) Write this information in an equation using x as the variable.

   ....

   ....

   ....

   (b) Solve the equation to determine the value of x.

   ....

   ....

   ....

   (c) How long is the rectangle?

   ....

   ....

   ....

4. The area of a rectangle is $(8x^2 + 2x)$ cm^2, and the length is $2x$ cm. Determine the width of the rectangle in terms of x, in its simplest form.

   ....

   ....

   ....

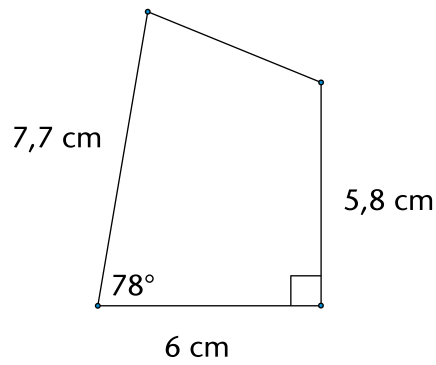
CONSTRUCTION OF GEOMETRIC FIGURES

Do not erase any construction arcs in these questions.

1. (a) Construct $D\hat{E}F = 56°$ with your ruler, pencil, and a protractor. Label the angle correctly.

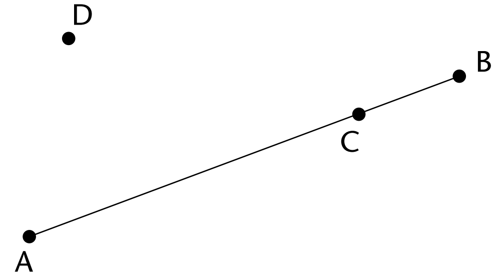
   (b) Bisect $D\hat{E}F$ using only a compass, ruler, and pencil (no protractor).

2. Here is a rough sketch of a quadrilateral (NOT drawn to scale):



Construct the quadrilateral accurately and full size below.

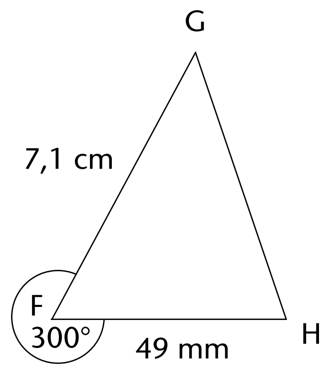
3. Using only a compass, ruler and pencil, construct:
   (a) A line through C perpendicular to AB
   (b) A line through D perpendicular to AB



4. Construct and label the following triangles and quadrilaterals:
    (a) Triangle ABC, where AB = 8 cm; BC = 5,5 cm and AC = 4,9 cm

    (b) Rhombus GHJK, where GH = 6 cm and $\widehat{G} = 50°$

5. Here is a rough sketch of triangle FGH (NOT drawn to scale):



Using a ruler, pencil, and protractor, construct and label the triangle accurately.

6. Construct an angle of 120° without using a protractor.

GEOMETRY OF 2D SHAPES

1. True or false: all equilateral triangles, no matter what size they are, have angles that equal 60°.

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2. (a) In a triangle, two of the angles are 35° and 63°. Calculate the size of the third angle.

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(b) In a quadrilateral, one of the angles is a right angle, and another is 80°. If the remaining two angles are equal to each other, what is the size of each?

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3. If triangle MNP has $\widehat{M} = 40°$ and $\widehat{N} = 90°$, what is the size of $\hat{P}$?

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4. Write definitions of the triangles in the table below.

Equilateral triangle	Isosceles triangle	Right-angled triangle

5. The following list gives the properties of three quadrilaterals, A, B and C.
   (a) Give the special names of each of shapes A, B and C.
   Quadrilateral A: The opposite sides are equal and parallel.

   ....

   Quadrilateral B: The adjacent sides are equal, while the opposite sides are not equal.

   ....

   Quadrilateral C: All of the angles are right angles.

   ....

   (b) What property must Quadrilateral A also have to make it a rhombus?

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   (c) What property must Quadrilateral A also have to make it a rectangle?

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6. Determine the size of $\hat{V}$. Show all steps of your working and give reasons.

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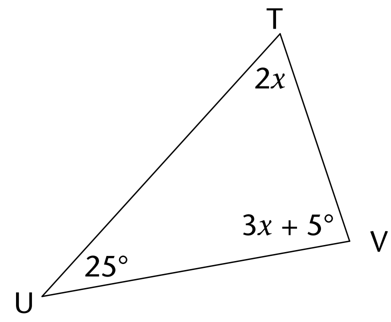

7. Determine the size of x. Give reasons.

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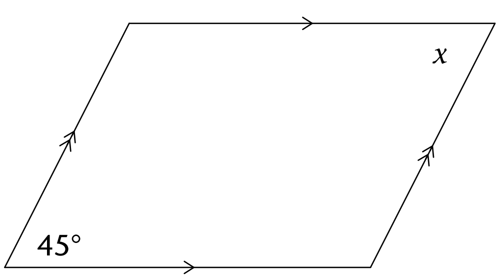
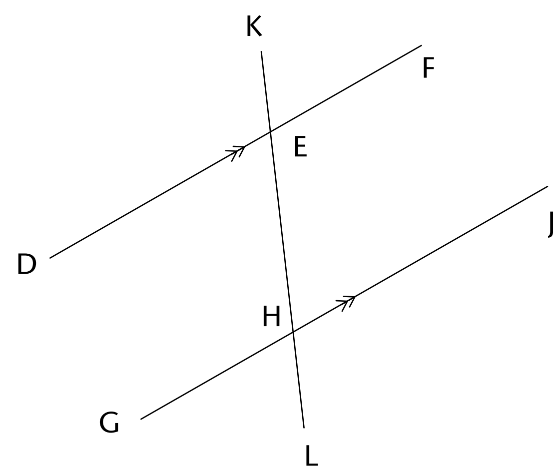


GEOMETRY OF STRAIGHT LINES

1. Study the diagram alongside:



   (a) Name an angle that is vertically opposite to $E\hat{H}G$.

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   (b) Name an angle that is corresponding to $E\hat{H}G$.

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   (c) Name an angle that is co-interior with $E\hat{H}G$.

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   (d) Name an angle that is alternate to $E\hat{H}G$.

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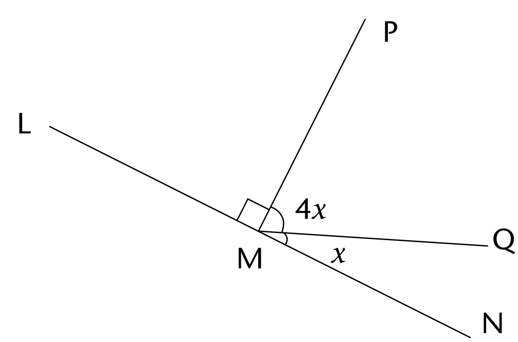
2. Determine the size of x in each of the following diagrams. Show all steps of working and give reasons.

   (a)



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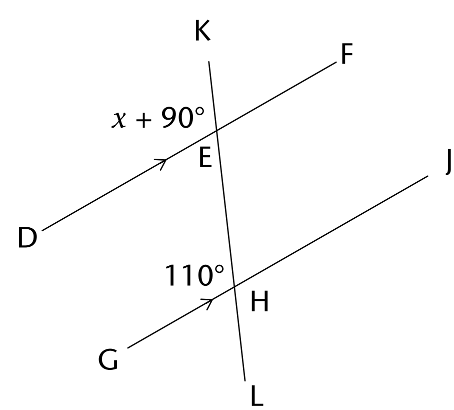
   (b)



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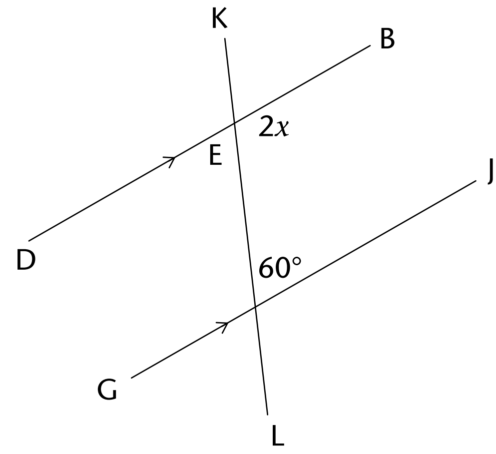
(c)



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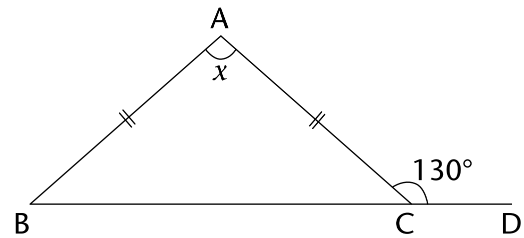
(d)



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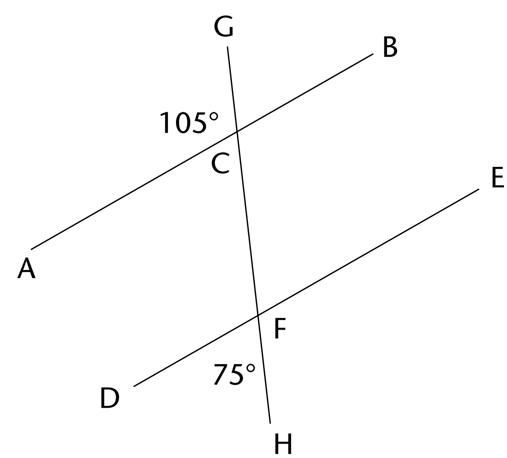
(e) Are line segments AB and DE parallel? Prove your answer.



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Assessment

In this section, the numbers in brackets at the end of a question indicate the number of marks the question is worth. Use this information to help you determine how much working is needed. The total number of marks allocated to the assessment is 75.

1. Simplify the following expressions:
   (a) $5x^2 - 6x^2 + 10x^2$ (1)

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   (b) $4(3x - 7) - 3(2 + x)$ (2)

   ....

   ....

   (c) $(-2a^2bc^3)^2 \div 4abcd$ (3)

   ....

   ....

   (d) $\dfrac{2x(3x - 15)}{3x}$ (3)

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   ....

   (e) $\sqrt[3]{108d^{15} \div 4d^6}$ (3)

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   (f) $2[3x^2 - (4 - x^2)] - [9 + (4x)^2]$ (3)

   ....

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2. Find the value of a if $b = 3$, $c = -4$ and $d = 2$:

(a) $a = b + c \times d$ (2)

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(b) $ab^2 = 2c - d \div 2$ (3)

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3. Solve the following equations:

(a) $-7x = 56$ (2)

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(b) $4(x + 3) = 16$ (2)

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4. Sipho, Fundiswa and Ntosh are brothers. Sipho earns Rx per month; Fundiswa earns R1 000 more than Sipho per month, and Ntosh earns double what Sipho earns. If you add their salaries together you get a total of R27 000.

(a) Write this information in an equation using x. (2)

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(b) Solve the equation to find how much Fundiswa earns per month. (2)

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5. Construct the following figure using only a pencil, ruler and compass. Do not erase any construction arcs.
   (a) An angle of 60° (2)

   (b) The perpendicular bisector of line VW, where VW = 10 cm (3)

   (c) Triangle KLM, where KL = 8,3 cm; LM = 5,9 cm and KM = 7 cm (4)

(d) Parallelogram EFGH, where E = 60°, EF = 4,2 cm and EH = 8 cm (4)

6. (a) What is/are the property/properties that make a rhombus <u>different</u> to a parallelogram? (1)

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(b) True or false: a rectangle is a special type of parallelogram. (1)

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7. Determine the size of x in each figure. Show all the necessary steps and give reasons.

(a) (3)

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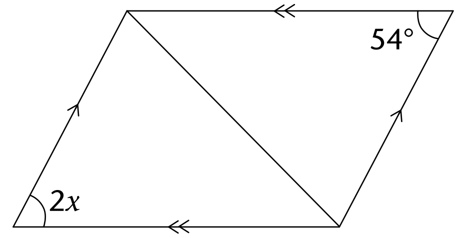

(b) (4)

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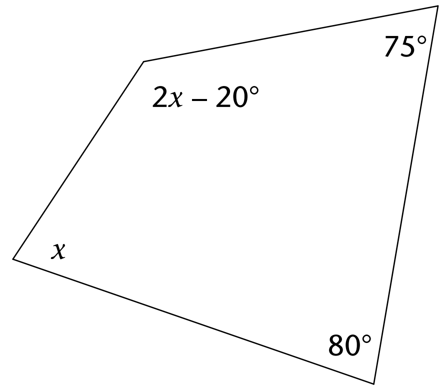

(c) (3)

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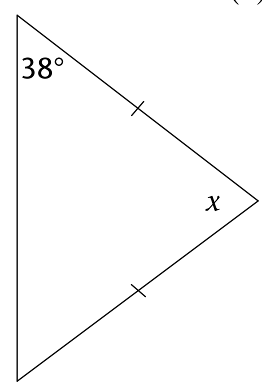
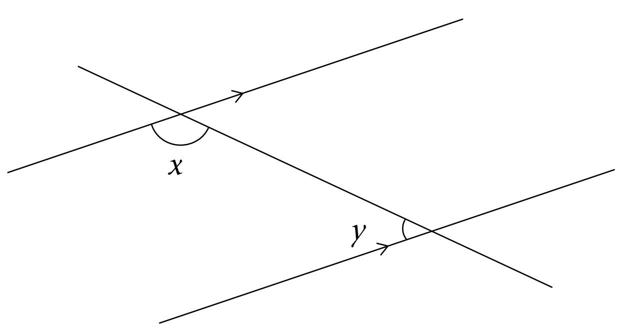


8. Study the following diagram. Then answer the questions that follow:



(a) Write down the correct word to complete the sentence: x and y form a pair of

. . . . angles. (1)

(b) Write down an equation that shows the relationship between angles x and y. (1)

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9. Determine the size of x, showing all necessary steps and giving reasons for all statements that use geometrical theorems:

(a) (4)

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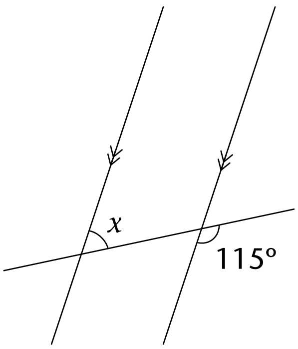

(b) (5)

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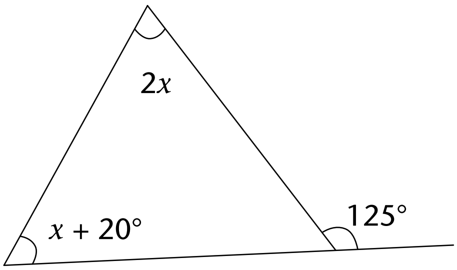

(c) (3)

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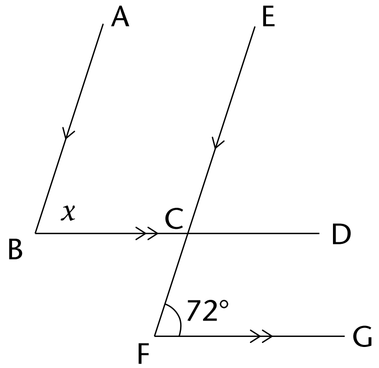
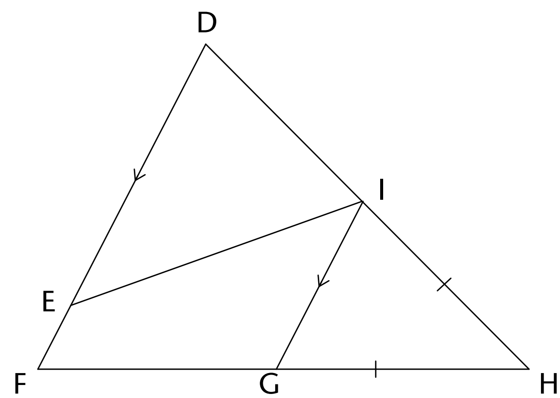


10. Consider the following diagram, in which it is given: $D\hat{E}I = 30°$, $DE = EI$, $DF \parallel IG$, and $GH = IH$.



(a) Determine, with reasons, the size of $\hat{H}$. (6)

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(b) Which of the following statements is correct? Explain your answer. (2)

(i) ΔDEI is similar to ΔGHI
(ii) ΔDEI is congruent to ΔGHI
(iii) We cannot determine a relationship between ΔDEI and ΔGHI since there is not enough information given.

Statement .... is correct because

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